

9-1

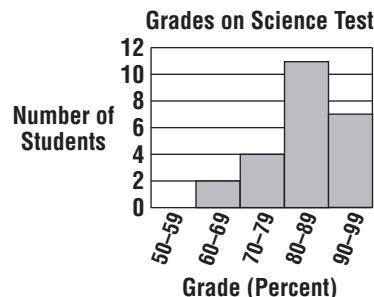
Histograms (pages 420–424)

Statistics involves collecting, organizing, and analyzing data. You can display data with a type of bar graph called a **histogram**. A histogram uses bars to display numerical data organized into equal intervals.

EXAMPLE

In the histogram shown at the right, how many students made a grade of 90–99 percent on the science test? How many made a grade of 50–59 percent?

The bar for a grade of 90–99 ends halfway between 6 and 8 on the vertical axis. So 7 students made an A. Categories that have a frequency of 0 have no bar. Since the category for 50–59 has no bar, there were 0 students who made a grade of 50–59 percent.



Try These Together

1. Refer to the histogram in Exercise 3. How many presidents were between the ages of 40 and 44 when inaugurated?
2. Refer to the histogram in Exercise 3. How large is each interval?

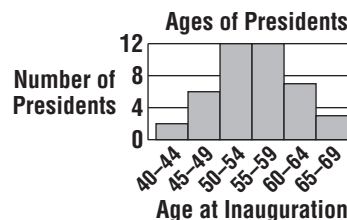
HINT: What is the height of the bar for the interval 40–44?

HINT: Count the number of ages each interval contains.

PRACTICE

3. Use the histogram at the right to answer each question.

- a. Which interval has the least number of presidents?
- b. Construct a frequency table from the data.



4. **Genealogy** Annabeth surveyed the students in her grade to find out how many CDs they each had. The results of her survey are shown in the table. Make a histogram of the data.

CDs per Student	
CDs	Students
1–10	16
11–20	20
21–30	28
31–40	8

5. **Standardized Test Practice** Refer to the histogram of test grades in the example above. Which grade interval was earned by the greatest number of students?

- A** 90–99 percent **B** 80–89 percent **C** 70–79 percent **D** 60–69 percent

Answers: 1. 2 2. 5 years 3a. 40–44 3b. See Answer Key 4. See Answer Key 5. B

9-2

Circle Graphs (pages 426–429)

A **circle graph** compares parts of a set of data to the whole set.

Drawing a Circle Graph	<ul style="list-style-type: none"> • If the data is given in numbers (rather than percents), first find the total number and find a ratio that compares each category to the total. • Multiply each ratio or percent by 360 degrees to find the number of degrees for that section of the graph. • Use a compass to draw a circle. Draw a radius. Use a protractor to draw any of the angles. From the new radius, use the protractor to draw the next angle, and repeat. • Label each section. Write each ratio as a percent. Title the graph.
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EXAMPLE

How many degrees will you draw in a circle graph to represent 25%?

Write the percent as a decimal: 0.25.

Multiply the decimal by 360 degrees: $0.25 \times 360 = 90$.

90 degrees represents 25% of the circle.

Try This Together

1. Use the table in Exercise 2 to find the number of degrees in the section of the circle graph that represents dogs in 1-person families.
HINT: Find 13% of 360.

PRACTICE

2. **Pets** The table shows the percent of dogs that lived with 1, 2, 3, and 4-person families in a recent year.
- Make a circle graph of the data.
 - Which family size owns about one-fifth of the dogs?

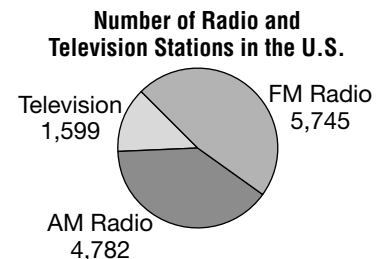
Number of People in Family	Percent of Dogs
1	13%
2	31%
3	21%
4	35%

3. **School** Suppose that in the United States there are 38,289,000 students in kindergarten through 8th grade; 16,299,000 students in 9th through 12th grade; and 16,228,000 students in college. Make a circle graph of this data.



4. **Standardized Test Practice** The circle graph shows the number of radio and television stations in the U.S. in 1999. About what percent of radio and television stations were AM radio stations?

- A** 47% **B** 55%
C 39% **D** 29%



Answers: 1. 47 degrees 2a. See Answer Key. 2b. 3-person families 3. See Answer Key. 4. C

9-3**Choosing an Appropriate Display** (pages 430–433)

When you choose what type of graph to use, ask yourself these questions:
What type of information is this? What do I want my graph to show?

	Display	Use
Choosing a Display	Bar Graph	shows the number of items in specific categories in the data using bars
	Circle Graph	compares parts of the data to the whole
	Histogram	shows the frequency of data that has been organized into equal intervals
	Line Graph	shows change over a period of time
	Line Plot	shows how many times each number occurs in the data
	Pictograph	shows the number of items in specific categories using symbols to represent a quantity
	Stem-and-Leaf Plot	lists all individual numerical data in a condensed form
	Table	may list all the data individually or by groups

EXAMPLE

Choose an appropriate type of display to compare people's annual salary to their number of years of education.

You could plot the salaries and years of education on a line graph.

You could examine the line graph to see if salaries increase with more education.

Try These Together

- Choose an appropriate type of display for the populations of five different cities in 2000.
- Choose an appropriate type of display for the votes received by four candidates in an election.

HINT: There are five categories which are not numerical.

HINT: Election results are often reported as percents of the whole.

PRACTICE

Choose an appropriate type of display for each situation.

- the numbers of students in your math class whose heights are 55 to 59 inches, 60 to 64 inches, 65 to 69 inches, and 70 to 74 inches
- students' grades on a math test and the numbers of hours they studied
- numbers of Americans who own 0, 1, 2, 3, or 4 or more cars

- Standardized Test Practice** What type of a display would you use to show the number of states with different numbers of national parks?

A histogram

B line plot

C circle graph

D line graph

Answers: 1–7. Sample answers are given. 1. bar graph 2. circle graph 3. histogram 4. line graph 5. circle graph or bar graph 6. A

9-4**Measures of Central Tendency** (pages 435–438)

Measures of central tendency use one number to describe a set of data.

Measures of Central Tendency

- The **mean** is the sum of the data divided by the number of items in the data set.
- The **median** is the number in the middle when you order the data from least to greatest. When there are two middle numbers, the median is the mean of those two.
- The **mode** is the number or numbers that occur most often.

EXAMPLE

Find the mean, median, and mode of the data. 11, 23, 47, 11, 25, 54

Find the total. Then divide by 6. $\frac{171}{6} = 28.5$

The mean is 28.5.

To find the median, arrange the data in order. 11, 11, 23, 25, 47, 54

There are two middle numbers, 23 and 25. The mean of 23 and 25 is

$\frac{23 + 25}{2}$ or 24. The median of the data is 24.

The number that appears most often is 11, which appears twice.

The mode is 11.

Try These Together

1. Find the mean, median, and mode of the data. 17, 15, 15, 12, 16

HINT: Find the total and divide by 5 to find the mean. Arrange in order to find the median.

2. Find the mean, median, and mode of the data. 3, 2, 3, 2, 3, 9, 5, 6, 4, 5, 2

HINT: There are two modes.

PRACTICE

Find the mean, median, and mode of each set of data. Round to the nearest tenth if necessary.

3. 58, 63, 57, 52, 58, 52, 52, 64

4. 110, 150, 142, 120, 113, 110, 123

5. 35, 35, 36, 32, 34, 33, 32, 31

6. 500, 1,000, 700, 1,000, 1,000, 1,200

7. **Employment** Kezia conducted a study to find out what the average wage was for high school students who were employed. The data she gathered is shown below. Find the mean, median, and mode of her data. Round to the nearest cent.

\$5.50 \$6.75 \$5.25 \$5.75 \$6.25 \$5.75 \$6.75 \$5.50 \$5.25 \$5.25



8. **Standardized Test Practice** The high temperatures in New York, NY, for one week in the summer were 80°F, 78°F, 80°F, 81°F, 85°F, 82°F, and 79°F. What was the median high temperature?

A 79°F

B 80°F

C 81°F

D 85°F

Answers: 1. 15; 15; 15 2. 4; 3; 2 and 3 3. 57; 57.5; 52 4. 124; 120; 110 5. 33.5; 33.5; 32 and 35 6. 900; 1,000; 1,000 7. \$5.80; \$5.63; \$5.25 8. B

9-5**Measures of Variation** (pages 442–445)

The spread of data is called the **variation**. One way to measure it is with the **range**, the difference between the greatest and least numbers in the set. With large sets of data, it is often helpful to separate the data into four equal parts called **quartiles**.

EXAMPLE

Find the range, median, upper and lower quartiles, and interquartile range for this set of data.

12, 12, 16, 14, 13, 13, 11, 15, 13, 15

Arrange the data in order and divide it into halves.

11, 12, 12, 13, 13, 13, 14, 15, 15, 16

The range is the difference between the greatest and least values.

$$16 - 11 = 5$$

The range is 5.

There are 2 middle numbers, 13 and 13, so the median is 13.

The median of the upper half of the data is 15, so 15 is the upper quartile.

The median of the lower half of the data is 12, so 12 is the lower quartile.

To find the interquartile range, subtract the lower quartile from the upper quartile. The difference is $15 - 12$, or 3. The interquartile range is 3.

Try These Together

- Find the range, median, and upper and lower quartiles for this set of data.
0, 5, 3, 3, 2, 5, 6, 4, 6, 9, 6
- Find the interquartile range for the set of data in Exercise 1.

HINT: First arrange the data in order.

HINT: Subtract the quartiles.

PRACTICE

Find the range, median, upper and lower quartiles, and interquartile range for each set of data.

3. 9, 2, 3, 8, 6, 1, 4, 6

4. 41, 45, 42, 42, 45, 46, 41, 43, 43

5. 75, 85, 75, 75, 85, 95, 96, 130, 78

6. 32, 16, 12, 21, 29, 19, 30, 25, 25, 26



7. **Standardized Test Practice** What is the interquartile range for a set of data whose upper quartile is 5.5 and whose lower quartile is 1.8?

A 7.3

B 9.9

C 3.7

D 1.9

Answers: 1. 9; 5; 6; 3 2. 3 3. 8; 5; 7; 2.5; 4.5 4. 5; 43; 45; 41.5; 3.5 5. 55; 85; 95.5; 75; 20.5 6. 20; 25; 29; 19; 10 7. C

9-6

Box-and-Whisker Plots (pages 446–449)

A **box-and-whisker plot** uses a number line to show the distribution of a set of data. The box is drawn around the quartile values, and the whiskers extend from each quartile to the extreme data points that are not outliers.

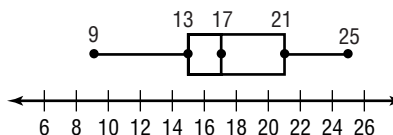
<p>Drawing a Box-and-Whisker Plot</p>	<ol style="list-style-type: none"> 1. Draw a number line that includes the least and greatest number in the data. 2. Mark the extremes, the median, and the upper and lower quartile above the number line. If the data has an outlier, mark the greatest value that is not an outlier. 3. Draw the box and the whiskers.
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EXAMPLE

Draw a box-and-whisker plot for this data: 18, 19, 16, 23, 25, 9, 10, 16

Arrange the data in order from least to greatest (9, 10, 16, 16, 18, 19, 23, 25). Draw a number line that includes the least and greatest numbers (9 and 25).

Mark the extremes (9 and 25), the median (17), the upper quartile (21), and the lower quartile (13) above the number line.



Draw the box and the whiskers.

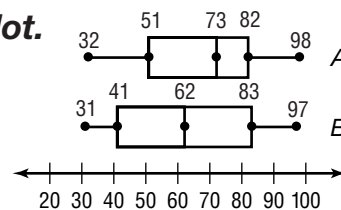
PRACTICE

Draw a box-and-whisker plot for each set of data.

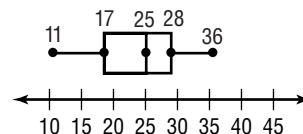
1. 283, 251, 225, 281, 290, 273, 204, 267
2. 102, 105, 80, 15, 90, 95, 106, 87, 80, 80, 105, 87, 85, 86
3. 27, 40, 30, 14, 19, 25, 27, 35, 31, 36, 39, 18, 30, 30, 35, 14

For Exercises 4–7, use the following box-and-whisker plot.

4. Which set of data is more spread out?
5. What is the interquartile range of class A's test scores?
6. Twenty-five percent of the students in class B scored below what average?



7. In general, which class scored higher on the test?
8. **Standardized Test Practice** Use the box-and-whisker plot at the right. Fifty-percent of the data are found between what two values?



A 25 and 36

B 28 and 36

C 11 and 28

D 17 and 36

Answers: 1–3. See Answer Key. 4. B 5. 31 6. 41% 7. A 8. A

9-7**Misleading Graphs and Statistics** (pages 450–453)

When dealing with statistics, be careful to identify when statistics are presented in a misleading way. Recall that there are three different measures of central tendency or types of averages. They are mean, median, and mode. These different values can be used to show different points of view.

EXAMPLES

The prices for sandwiches at a fast-food restaurant are \$0.99, \$1.29, \$3.39, \$0.99, \$0.99, \$3.19, \$2.49, \$0.99, \$3.19, \$1.49, \$2.79, \$2.49, and \$1.49.

A Find the mean, median, and mode of the prices.

$$\text{Mean} \quad \frac{\text{sum of values}}{\text{number of values}} = \frac{\$25.77}{13} \approx \$1.98$$

Median *Ordering the prices from least to greatest, the middle price is \$1.49.*

Mode *\$0.99*

B Which average would the restaurant use to encourage people who want to save money to eat at their restaurant? Explain.

People who want to save money would like more inexpensive sandwiches. Therefore, the restaurant would use the mode since it is the least of the averages.

C Which average or averages would be more representative of the data?

Since the mode is the least value of the thirteen values, it is not as representative as the mean or median.

PRACTICE

For Exercises 1–5, use the list of times, in minutes, it took two different groups of students to complete a homework assignment.

Group 1: 60, 45, 40, 30, 25, 22, 20, 20, 20, 15

Group 2: 45, 40, 32, 30, 25, 22, 18, 18, 15, 15

1. What is the mean, median, and mode of the times for Group 1?
2. If the two groups are competing to see who finished faster, what average is most favorable for Group 1?
3. What is the mean, median, and mode of the times for Group 2?
4. Which group was faster in completing the assignment? Explain.



- 5. Standardized Test Practice** In the following set of data, which value is smallest?

Miles traveled per day on a trip: 420, 125, 375, 283, 198, 420, 632, 480

- A** mean **B** median **C** mode **D** none of these

Answers: 1. 29.7; 23.5; 20 2. mode 3. 13; 23.5; 18 4. Group 2; the mean time was much lower for Group 2. 5. C

9-8**Matrices** (pages 454–457)

One way to organize information is by using a **matrix**. A matrix is a rectangular arrangement of numbers in rows and columns. Each number in a matrix is called an **element** of the matrix.

Adding and Subtracting Matrices

- You can add or subtract matrices that have the same number of rows and the same number of columns.
- Add or subtract matrices by adding or subtracting the corresponding elements.

EXAMPLES

A Add $\begin{bmatrix} 2 & 0 & -1 \\ 1 & -2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 1 \\ -1 & 3 & 4 \end{bmatrix}$.

Both matrices have 2 rows and 3 columns, so you can add them by adding the corresponding elements.

$$\begin{bmatrix} 2+1 & 0+5 & -1+1 \\ 1+(-1) & -2+3 & 3+4 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 0 \\ 0 & 1 & 7 \end{bmatrix}$$

B Add $\begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$.

The first matrix has 3 rows and 2 columns, but the second matrix has 2 rows and 3 columns. It is impossible to add these matrices.

Try These Together

1. Add. If there is no sum, write *impossible*.

$$\begin{bmatrix} 5 & 2 & 4 \\ 1 & 6 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 8 & 2 & 1 \end{bmatrix}$$

HINT: $5 + 1 = 6$, $2 + 0 = 2$, and so on.

2. Subtract. If there is no difference, write *impossible*.

$$\begin{bmatrix} 5 & 3 \\ 8 & 2 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 0 \\ 0 & 5 \end{bmatrix}$$

HINT: Do these matrices have matching numbers of rows and columns?

PRACTICE

Add or subtract. If there is no sum or difference, write impossible.

3. $\begin{bmatrix} 1 & 5 \\ -8 & -1 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 8 \\ 9 & 11 \end{bmatrix} + [3 \ 7 \ 15]$

5. **Population** The table shows the populations of Montana and Idaho in 1980, 1990, and 2000. Write a matrix for the data.

Population (thousands)		
Year	Montana	Idaho
1980	787	944
1990	799	1,007
2000	902	1,294



6. **Standardized Test Practice** Find the sum of $\begin{bmatrix} 8 & -3 & 5 \\ 0 & -1 & 2 \end{bmatrix}$ and

$$\begin{bmatrix} 3 & 6 & -7 \\ -8 & 5 & 3 \end{bmatrix}$$

A $\begin{bmatrix} 11 & 3 & -2 \\ -8 & 4 & 5 \end{bmatrix}$

B $\begin{bmatrix} 5 & -9 & 12 \\ 8 & -6 & -1 \end{bmatrix}$

C $\begin{bmatrix} -11 & -3 & 2 \\ 18 & 6 & 1 \end{bmatrix}$

D $\begin{bmatrix} -5 & 9 & -12 \\ -18 & 4 & 5 \end{bmatrix}$

Answers: 1–3. See Answer Key. 4. impossible 5. See Answer Key. 6. A

