

10-1

Simplifying Algebraic Expressions (pages 469–473)

Simplifying Algebraic Expressions	The expressions $3(x + 4)$ and $3x + 12$ are equivalent expressions , because no matter what x is, these expressions have the same value.
	When a plus sign separates an algebraic expression into parts, each part is called a term . The numerical part of a term that contains a variable is called the coefficient of the variable. Like terms are terms that contain the same variables, such as $4x$ and $5x$. A term without a variable is called a constant . Constant terms are also like terms.
	An algebraic expression is in simplest form if it has no like terms and no parentheses. You can use the Distributive Property to combine like terms. This is called simplifying the expression .

EXAMPLES

A Use the Distributive Property to rewrite the expression $8(x + 5)$.

$$\begin{aligned} 8(x + 5) &= 8(x) + 8(5) \\ &= 8x + 40 \quad \text{Simplify.} \end{aligned}$$

B Identify the terms, like terms, coefficients, and constants in the expression $5y - 4 + 6y$.

terms: $5y, -4, 6y$ coefficients: $5, -4$ and 6
like terms: $5y$ and $6y$ constants: -4

C Simplify $-3t + 11 - 4t$.

$$\begin{aligned} -3t \text{ and } -4t \text{ are like terms.} \\ -3t + 11 - 4t &= -3t - 4t + 11 \\ &= [-3 + (-4)]t + 11 \\ &= -7t + 11 \end{aligned}$$

PRACTICE

Use the Distributive Property to rewrite each expression.

1. $2(y + 11)$ 2. $3(2b - 3)$ 3. $-6(10r + 3)$

Identify the terms, like terms, coefficients, and constants in each expression.

4. $4 - 3r + r - 2$ 5. $-2t - 3 + 11 - 4t$ 6. $16y - 5 + 2y - y$

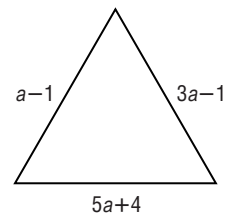
Simplify each expression.

7. $6x - 2x$ 8. $4y + 7 + 12y$ 9. $6r - 2r + 1$



10. Standardized Test Practice Which expression represents the perimeter of the figure at the right?

- A** $5a + 2$ **B** $a + 6$
C $9a + 6$ **D** $9a + 2$



Answers: 1. $2y + 22$ 2. $6b - 9$ 3. $-60r - 18$ 4. terms: $4, -3r, r, -2$; like terms: $4, -2$ and $-3r, r$; coefficients: $4, -3, 1, -2$; constant: $4, -2$ 5. terms: $-2t, -4t$; like terms: $-2t, -4t$ and $-3, 11$; coefficients: $-2, -3, 11, -4$; constant: $-3, 11$ 6. terms: $16y, 2y, -y$; like terms: $16y, 2y, -y$; coefficients: $16, -5, 2, -1$; constant: -5 7. $4x$ 8. $16y + 7$ 9. $4r + 1$ 10. D

10-2**Solving Two-Step Equations** (pages 474–477)

In some algebraic equations, two operations, such as addition and multiplication, are performed on a variable. An example is $2x + 1 = 5$. Such equations are known as **two-step equations**.

Solving Two-Step Equations	Solve $2x + 1 = 5$.	First, use inverse operations to “undo” any addition or subtraction operations. Then use inverse operations to “undo” any multiplication or division operations. Notice that this is in the opposite order from the order of operations.
	$2x + 1 - 1 = 5 - 1$	
	$2x = 4$	
	$\frac{2x}{2} = \frac{4}{2}$	
	$x = 2$	

EXAMPLE

Solve $8 - 3b = 26$.

$$8 - 3b = 26$$

$$8 - 8 - 3b = 26 - 8$$

Subtract 8 from each side.

$$-3b = 18$$

$$\frac{-3b}{-3} = \frac{18}{-3}$$

Divide each side by -3 .

$$b = -6$$

The solution is -6 . Be sure to check your answer.

Try These Together

Solve each equation. Check your solution.

1. $2d + 10 = 20$

2. $3f - 15 = 12$

3. $9 - 4t = 25$

HINT: Remember to “undo” operations.

PRACTICE

Solve each equation. Check your solution.

4. $30 = 5p + 25$

5. $2x - 3 = 9$

6. $8g - 24 = 8$

7. $17 - 12r = 41$

8. $64 = 4s + 16$

9. $50 = 6z - 10$

10. $\frac{n}{3} + 8 = 11$

11. $\frac{m}{15} - 2 = 0$

12. $5.8 - 3a = 14.8$

- 13. Entertainment** At an amusement park, admission for the first 5 people in Bob’s family cost \$20 per person, or \$100 total. The remaining people in the group got in at a lower rate. If Bob’s family is a group of 8, and the total cost was \$145, how much was the admission, per person, for the other three people?



- 14. Standardized Test Practice** Find n if $4n - 16 = 36$.

A 14

B 12

C 13

D 15

Answers: 1. 5 2. 9 3. -4 4. 1 5. 6 6. 4 7. -2 8. 12 9. 10 10. 9 11. 30 12. -3 13. \$15 each 14. C

10-3

Writing Two-Step Equations (pages 478–481)

Some verbal sentences translate to two-step equations. There are many real-life situations in which you start with a given amount and then increase it at a certain rate. These situations can be represented by two-step equations.

EXAMPLE

Translate and solve the equation.

Seven less than twice a number is 15. Find the number.

Words	<i>Seven less than twice a number is 15.</i>	
Variables	<i>Let $n =$ the number.</i>	
Equation	$2n - 7 = 15$	<i>Write the equation.</i>
	$2n - 7 + 7 = 15 + 7$	<i>Add 7 to each side.</i>
	$2n = 22$	<i>Simplify.</i>
	$\frac{2n}{2} = \frac{22}{2}$	<i>Divide each side by 2.</i>
	$n = 11$	<i>Simplify.</i>

Therefore, the number is 11.

PRACTICE

Translate each sentence into an equation. Then find each number.

- Eight less than six times a number is equal to -2 .
- The quotient of a number and 4, plus 2, is equal to 10.
- The difference between four times a number and thirteen is 15.
- If 11 is increased by three times a number, the result is 2.
- Six times a number minus three times the number plus 1 is -5 .

Solve each problem by writing and solving an equation.

- Kyle wants to save for a new pair of shoes. The shoes cost \$109.99. He already has \$85 in his savings account. How much more does he need to save?
- Kate has two sisters. Kate is twice as old as one of her sisters and five years older than her other sister. If the sum of their ages is 35, how old is each sister?



- Standardized Test Practice** Brad spent \$143.10 dollars at a sporting goods store. If the sales tax was 6%, which of the following equations can be used to find the amount (b) before the sales tax?

- A** $b + 0.06b = 143.10$ **B** $b + 6b = 143.10$
C $143.10 + b(0.06) = b$ **D** $b + 0.06 = 143.10$

Answers: 1. $6n - 8 = -2; n = 1$ 2. $\frac{7}{n} + 2 = 10; n = 32$ 3. $4n - 13 = 15; n = 7$ 4. $11 + 3n = 2; n = -3$ 5. $6n - 3n + 1 = -5; n = -2$ 6. let $n =$ what Kyle needs to save; $85 + n = 109.99; n = 24.99$; Kyle needs to save \$24.99 7. let $x =$ Kate's age; $\left(\frac{2}{1}x\right) + (x) + (-5) = 35; x = 16$; Kate is 16 and her sisters are 8 and 11. 8. A

10-4**Solving Equations with Variables on Each Side** (pages 483–487)

Some equations have variables on each side of the equals sign. To solve these equations, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variables on one side of the equals sign. Then solve the equation.

EXAMPLE

Solve $24 - 2y = 4y$. Check your solution.

$$24 - 2y = 4y \quad \text{Write the equation.}$$

$$24 - 2y + 2y = 4y + 2y \quad \text{Add } 2y \text{ to each side.}$$

$$24 = 6y \quad \text{Simplify.}$$

$$4 = y \quad \text{Mentally divide each side by 6.}$$

To check your solution, replace y with 4 in the original equation.

Check $24 - 2y = 4y$ Write the equation.

$$24 - 2(4) \stackrel{?}{=} 4(4) \quad \text{Replace } y \text{ with 4.}$$

$$16 = 16 \quad \text{The sentence is true.}$$

The solution is 4.

PRACTICE

Solve each equation. Check your solution.

1. $6x + 4 = 7x$

2. $13k - 12 = 9k$

3. $2p = p - 21$

4. $8 - 3r = 5r$

5. $6 - 5j = 2j - 8$

6. $s - 2 = 3s + 8$

7. $16.4 - d = 3d$

8. $6.1\ell + 24 = 9.3\ell$

9. $5m - 26 = -7m - 34$

10. $7 - 3c = 4 + 3c$

11. $9y + 1.2 = -16.8 - 21y$

12. $1 - 4x = 6x + 13$

13. $\frac{3}{4}k - 6 = \frac{1}{4}k + 1$

14. $2 - \frac{1}{6}m = \frac{1}{3}m + 7$

Define a variable and write an equation to find each number. Then solve.

15. Three times a number is 21 more than six times a number. What is the number?

16. Nine less than twice a number equals three times the number plus six. What is the number?

17. **Standardized Test Practice** Rental car company A charges \$36 a day plus \$0.25 per mile. Rental car company B charges \$21 a day plus \$0.35 per mile. Which equation can be used to find the number of miles for which the companies' plans cost the same?

A $36 + 0.25m = 21 + 0.35m$

B $36 + 0.35m = 21 + 0.25m$

C $36m + 0.25 = 21m + 0.35$

D $(36 + 0.25)m = (21 + 0.35)m$

Answers: 1. $x = 4$ 2. $k = 3$ 3. $p = -21$ 4. $r = 1$ 5. $j = 2$ 6. $s = -5$ 7. $d = 4.1$ 8. $\ell = 7.5$ 9. $m = -\frac{3}{2}$ 10. $c = 0.5$ 11. $y = -0.6$ 12. $x = -1.2$ 13. $k = 14$ 14. $m = -10$ 15. let $n =$ number; $3n = 21 + 6n$; $n = -7$ 16. let $n =$ number; $2n - 9 = 3n + 6$; $n = -15$ 17. A

10-5**Inequalities** (pages 492–495)

A mathematical sentence that contains $>$ or $<$ is called an **inequality**. When used to compare a variable and a number, inequalities can describe a range of values. Some inequalities use the symbols \leq or \geq . The symbol \leq is read *is less than or equal to*, while the symbol \geq is read *is greater than or equal to*.

Common Phrases and Corresponding Inequalities			
$<$	$>$	\leq	\geq
<ul style="list-style-type: none"> • is less than • is fewer than 	<ul style="list-style-type: none"> • is greater than • is more than • exceeds 	<ul style="list-style-type: none"> • is less than or equal to • is no more than • is at most 	<ul style="list-style-type: none"> • is greater than or equal to • is no less than • is at least

EXAMPLES

- A** Write an inequality for the sentence. Then graph the inequality on a number line.

Children 5 years of age and under are admitted free.

Let $c =$ child's age
 $c \leq 5$



To graph the inequality, place a closed circle at 5. Then draw a line and an arrow to the left.

- B** For the given value, state whether the inequality is *true* or *false*.

$$13 - x > 6, x = 4$$

$$13 - x > 6$$

$$13 - 4 > 6$$

$$9 > 6$$

Write the inequality.

Replace x with 4.

Simplify.

Since 9 is greater than 6, $13 - x > 6$ is true.

Try These Together

- Write an inequality for the sentence. More than 20 students must sign up in order to go on the field trip.
- For the given value, state whether the inequality is *true* or *false*.
 $t + 5 < 11, t = 8$

PRACTICE

Write an inequality for each sentence.

- You must sell at least 25 candy bars to qualify for a prize.
- No more than 4 students at each activity.

For the given value, state whether each inequality is true or false.

5. $7d \geq 28, d = 4$

6. $15 - y < 3, y = 6$

7. $9 \leq a + 1, a = 12$

Graph each inequality on a number line.

8. $m > 8$

9. $h \geq 22$

10. $b < 1$

- 11. Standardized Test Practice** Which inequality represents *a number is no more than 34*.

A $x < 34$

B $x > 34$

C $x \leq 34$

D $x \geq 34$

Answers: 1. $s < 20$ 2. false 3. $c \geq 25$ 4. $s \leq 4$ 5. true 6. false 7. true 8-10. See Answer Key. 11. C

10-6**Solving Inequalities by Adding or Subtracting** (pages 496–499)

An **inequality** is a mathematical sentence that compares quantities using symbols like $>$ and $<$ instead of an equals sign. Inequalities may have many solutions, which can be written as a set of numbers or graphed on a number line.

Addition and Subtraction Properties of Inequality

Words When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols For all numbers a , b , and c ,

- if $a > b$, then $a + c > b + c$ and $a - c > b - c$.
- if $a < b$, then $a + c < b + c$ and $a - c < b - c$.

Examples

$2 > 3$	$3 < 8$
$2 + 5 > -3 + 5$	$3 - 4 < 8 - 4$
$7 > 2$	$-1 < 4$

These properties are also true for $a \geq b$ and $a \leq b$.

EXAMPLE

Solve $n + 10 \leq 12$ and graph the solution on a number line.

$$n + 10 \leq 12 \quad \text{Write the inequality.}$$

$$n + 10 - 10 \leq 12 - 10 \quad \text{Subtract 10 from each side.}$$

$$n \leq 2 \quad \text{Simplify.}$$

All values of x that are less than or equal to 2 are solutions to the inequality. This is indicated by a closed circle on the number line at 2, and an arrow going to the left.

**Try These Together**

Solve each inequality and check your solution. Then graph the solution on a number line.

1. $y - 5 < 3$

2. $14 \leq 9 + x$

3. $f + 8 < 10$

HINT: When graphing, use a closed circle for \leq or \geq and an open circle for $<$ or $>$.

PRACTICE

Solve each inequality and check your solution. Then graph the solution on a number line.

4. $4 \leq g - 3$

5. $h - 1 \leq 2$

6. $6 + q < 16$

7. $5 + k \leq 11$

8. $m - 8 > 1$

9. $a + 9 \geq 12$

10. Standardized Test Practice Solve the inequality $x + 4 \leq 7$.

A $x \leq 28$

B $x \leq 11$

C $x \leq 3$

D $x \leq 9$

Answers: 1–9. See Answer Key for graphs. 1. $y < 8$ 2. $x < 8$ 3. $f < 2$ 4. $g > 7$ 5. $h \leq 3$ 6. $q < 10$ 7. $k \leq 6$ 8. $m > 9$ 9. $a \geq 3$ 10. C

10-7**Solving Inequalities by Multiplying or Dividing** (pages 500–504)

You can solve inequalities that have rational numbers in them the same way you solved inequalities with integers.

Solving Inequalities	<p>Use the same steps to solve an inequality as you use to solve an equation, with this one exception.</p> <ul style="list-style-type: none"> When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be reversed for the inequality to remain true.
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EXAMPLES**A** Solve $-3x < 12$.

$$\begin{aligned} -3x &< 12 \\ \frac{-3x}{-3} &> \frac{12}{-3} && \text{Divide each side by } -3. \\ x &> -4 \end{aligned}$$

Since you divided each side by -3 , the direction of the inequality symbol must be reversed. The solution is $x > -4$.

B Solve $\frac{y}{2} + 8 > 0$.

$$\begin{aligned} \frac{y}{2} + 8 &> 0 \\ \frac{y}{2} + 8 - 8 &> 0 - 8 && \text{Subtract 8 from each side.} \\ \frac{y}{2} &> -8 \\ 2\left(\frac{y}{2}\right) &> 2(-8) && \text{Multiply each side by 2.} \\ y &> -16 \end{aligned}$$

The solution to the inequality is $y > -16$.

Try These Together1. Solve $-7c > -21$.

HINT: Will the solution have a $>$ sign or a $<$ sign?

2. Solve $j + 0.06 < 4.5$.

HINT: Solve by subtracting 0.06 from each side.

PRACTICE**Solve each inequality.**

3. $6p \geq 3$

4. $-15 \leq -\frac{x}{5}$

5. $-8q > \frac{1}{2}$

6. $\frac{k}{9} - 5 \geq -9$

7. $\frac{m}{3} < 9\frac{1}{4}$

8. $5 + 5v > 52$

9. $-16a + 19 \geq 17\frac{1}{3}$

10. $-2z + 6 \leq 4$

11. $\frac{3n}{2} > 9$

**12. Standardized Test Practice** Solve $\frac{s}{3} - 8 > 4$.

A $s > 36$

B $s < 36$

C $s > -36$

D $s < -36$

Answers: 1. $c < 3$ 2. $j < 4.44$ 3. $p \geq \frac{2}{3}$ 4. $x \leq 75$ 5. $q < -\frac{16}{3}$ 6. $k \geq -36$ 7. $m < 27\frac{4}{3}$ 8. $v > 9\frac{5}{2}$ 9. $a \leq \frac{48}{5}$ 10. $z \geq 1$ 11. $n > 6$ 12. A

10**Chapter 10 Review****Algebra at the Zoo**

Substitute the values in the box into each problem below and solve.
Write your solution in the blank to the left of the problem.

 = 5	 = 3	 = 2	 = x	 = 4
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_____ 1.  \div  = 

_____ 2. $\left(\text{elephant}\right)\left(\text{monkey}\right) = \text{giraffe} + \text{camel}$

_____ 3. $\left(\text{giraffe}\right)\left(\text{monkey} + \text{giraffe}\right) = \text{elephant}$

_____ 4. $\left(\text{camel}\right)\left(\text{monkey} - \text{giraffe}\right) < \text{tiger} - 2\left(\text{giraffe}\right)$

Draw a square with a side of length  inches.

_____ 5. Find the area of the square.

_____ 6. Find the perimeter of the square.

Answers are located on page 110.