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11-1 Sequences (pages 511–515)

A **sequence** is a list of numbers in a certain order. Each number is called a **term** of the sequence. In an **arithmetic** sequence, the difference between any two consecutive terms is the same. This difference is called the **common difference**. In a **geometric sequence**, the consecutive terms of a sequence are formed by multiplying by a constant factor called the **common ratio**.

EXAMPLES

A Identify the pattern in 22, 19, 16, 13, 10, … and write the next five terms.

Try 19 − 22 = −3. If you add −3 to 19, do you get the next term, 16? Yes, and this pattern continues, so this is an arithmetic sequence with a common difference of −3. The next five terms are 7, 4, 1, −2, and −5.

B Identify the pattern in 20, 10, 5, 2.5, 1.25, … and write the next three terms.

There is no common difference. What can you multiply 20 by to get 10? 0.5. Does this common ratio continue? Yes, so this geometric sequence has a common ratio of 0.5. The next three terms are 0.625, 0.3125, and 0.15625.

Try These Together

1. State whether 0, 3, 6, 9, … is *arithmetic*, *geometric*, or *neither*. Then find the next three terms.
HINT: What can you add to each term to give you the next term?

2. State whether $-7, -6\frac{1}{2}, -6, -5\frac{1}{2}, \ldots$ is *arithmetic*, *geometric*, or *neither*. Then find the next three terms.
HINT: What can you add to each term to give you the next term?

PRACTICE

State whether each sequence is* arithmetic, geometric, *or* neither. *If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of each sequence.

3. $-3, -1, -\frac{1}{3}, -\frac{1}{9}, \ldots$
4. $-3, -2, 0, 3, \ldots$
5. $88, 93, 99, 106, \ldots$

6. $80, -40, 20, -10, \ldots$
7. $4, -3, -10, -17, \ldots$
8. $8, 8\frac{2}{3}, 9\frac{1}{3}, 10, \ldots$

9. Fitness Hank wants to increase the number of push-ups he does each day by 3. If on the first day he does 2, how many will he try to do on the 10th day?

10. Standardized Test Practice What is the next term in the sequence 1.3, 1.7, 2.1, 2.5, …?

A 3.3 **B** 3.1 **C** 2.9 **D** 2.7

Answers: 1. arithmetic; 12, 15, 18 **2.** arithmetic; $-5, -4\frac{1}{2}, -4$ **3.** geometric; $\frac{1}{3}$; $-\frac{1}{27}, -\frac{1}{81}, -\frac{1}{243}$ **4.** neither; 7, 12, 18 **5.** neither; 114, 123, 133 **6.** geometric; $-\frac{1}{2}$; $5, -2\frac{1}{2}, 1\frac{1}{4}$ **7.** arithmetic; −7; −24, −31, −38 **8.** arithmetic; $\frac{2}{3}$; $10\frac{2}{3}, 11\frac{1}{3}, 12$ **9.** 29 **10.** C

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11-2 Functions (pages 517–520)

A relationship where one thing depends upon another is called a **function**. In a function, one or more operations are performed on one number to get another number. So, the second number depends on, or is a function of, the first number. The value of $f(x)$ (which you say as "function of x" or "f of x") depends on the value of x.

Finding Values for Functions	You can organize the input (original number), rule (the operations performed on the input), and the output (the value of the function) into a **function table** like this one. Input or **domain** | Rule | Output or **range** x | $2x + 1$ | $f(x)$ The domain contains all the values of x, and the range contains all the values of $f(x)$.

Input or **domain**	Rule	Output or **range**
x	$2x + 1$	$f(x)$

EXAMPLE

Complete the function table at the right.

Replace x in the rule with each input value.
The rule, 2x + 1, is 2(0) + 1 or 1 for an input of 0.
Put the simplified value for f(x) in the output column.
Repeat these same steps for the input values of −1, 1, and 2.

Input x	Rule $2x + 1$	Output $f(x)$
−1	2(−1) + 1	−1
0	2(0) + 1	1
1	2(1) + 1	3
2	2(2) + 1	5

PRACTICE

1. Complete this function table.

x	$-3x$	$f(x)$
−2		
0		
0.5		
2		
4		

Find each function value.

2. $f(6)$ if $f(x) = x - 3$

3. $f(0.5)$ if $f(x) = 0.5x + 1$

4. $f(3.2)$ if $f(x) = x^2 - 2$

5. $f(12)$ if $f(x) = -x - 3$

6. $f(-4)$ if $f(x) = x + 5$

7. $f(0)$ if $f(x) = x + 5$

8. **Standardized Test Practice** If $f(x) = 2x^2 + 20$, find $f(-3)$.

A 2 **B** 38 **C** 56 **D** 236

Answers: **1.** 6, 0, −1.5, −6, −12 **2.** 3 **3.** 1.25 **4.** 8.24 **5.** −15 **6.** 1 **7.** 5 **8.** B

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11-3 Graphing Linear Functions (pages 522–525)

A function for which the graphs of the solutions form a straight line is called a **linear function**.

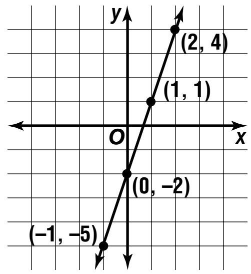
Graphing a Linear Function	To graph a linear function, begin by making a function table. List at least three values for *x*. Graph each ordered pair. Connect the points with a straight line. Add arrows to the ends of the line to show that the line continues indefinitely.

EXAMPLE

Graph the function $y = 3x - 2$.

Choose some values for x, and find the matching values for y. Make a table to show the ordered pairs.

x	$3x - 2$	y	(x, y)
−1	3(−1) − 2	−5	(−1, −5)
0	3(0) − 2	−2	(0, −2)
1	3(1) − 2	1	(1, 1)
2	3(2) − 2	4	(2, 4)



Then graph the ordered pairs from your table. Draw the line that joins these points. This line is the graph of $y = 3x - 2$.

Try These Together

1. Graph the function $y = 3x$.
   HINT: Make a function table for the x-values of −1, 0, 1, 2.

2. Graph the function $y = 6 - x$.
   HINT: Make a function table for the x-values of −1, 0, 2, 6.

PRACTICE

Graph each function.

3. $y = \frac{x}{2} + 3$
4. $y = x - 10$
5. $y = -x$
6. $y = \frac{1}{2}x + 4$
7. $y = 2x + 3$
8. $y = 5 - 2x$

9. **Standardized Test Practice** If it costs 25 cents to manufacture an eraser, how much would it cost to manufacture 10? Find the ordered pair that would represent this on a linear graph.

A (10, \$2.50) **B** (10, \$5) **C** (\$2.5, 8) **D** (2, \$25)

Answers: 1–8. See Answer Key. **9.** A

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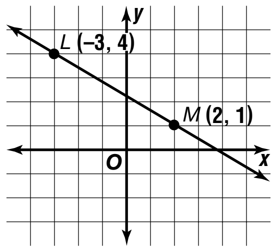
11-4 The Slope Formula (pages 526–529)

You can find the slope of a line by using the coordinates of any two points on the line. The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

EXAMPLE

Find the slope of the line that passes through $L(-3, 4)$ and $M(2, 1)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Definition of slope*

$m = \frac{1 - 4}{2 - (-3)}$ *$(x_1, y_1) = (-3, 4)$* *$(x_2, y_2) = (2, 1)$*

$m = \frac{-3}{5}$ *Simplify.*

PRACTICE

Find the slope of the line that passes through each pair of points.

1. $P(2, 2), Q(-3, -3)$ **2.** $R(-8, 9), S(2, 1)$ **3.** $X(-4, -5), Y(-8, -2)$

4. $M(3, 7), N(9, 7)$ **5.** $G(0, 0), H(7, 6)$ **6.** $V(13, -11), W(-2, 21)$

7. $P\left(\frac{1}{5}, -\frac{1}{8}\right), Q\left(3\frac{1}{5}, \frac{7}{8}\right)$ **8.** $R\left(\frac{3}{4}, \frac{1}{4}\right), S\left(1\frac{3}{4}, 3\frac{1}{4}\right)$ **9.** $J(4.5, -2.5), K(-6.5, -1.5)$

For Exercises 16 and 17, use the following information.

Caroline sells shirts for the pep club. After 3 shirts were sold, she had \$45. After 6 shirts were sold, she had \$90. After 7 shirts were sold, she had \$105.

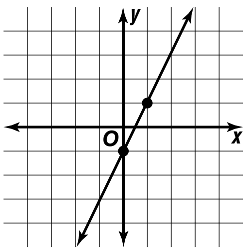
10. Graph the information with the number of shirts on the horizontal axis and the profit in dollars on the vertical axis. Draw a line through the points.

11. What is the slope of the graph?

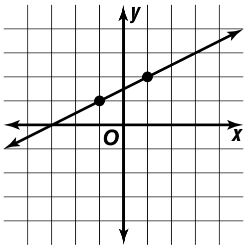
12. What does the slope of the graph represent?

13. **Standardized Test Practice** Which graph has a slope of $\frac{1}{2}$?

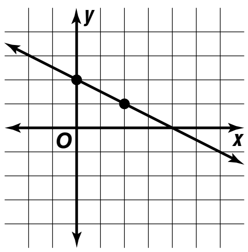
A



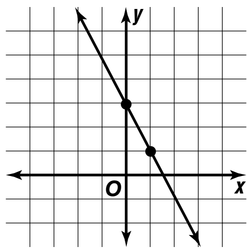
B



C



D



Answers: 1. 1 **2.** $-\frac{4}{5}$ **3.** $-\frac{3}{4}$ **4.** 0 **5.** $\frac{6}{7}$ **6.** $-\frac{32}{15}$ **7.** $\frac{1}{3}$ **8.** 3 **9.** $-\frac{1}{11}$ **10.** See Answer Key. **11.** 15 **12.** price per shirt **13.** B

11-5 Slope-Intercept Form (pages 533–536)

An equation of a line can be written in the form $y = mx + b$. This is called the **slope-intercept form**, where m is the slope of the line and b is the y-intercept of the line. For example, in the equation $y = -3x + (-2)$, the slope is –3 and the y-intercept is –2.

EXAMPLES

State the slope and the y-intercept of the graph of each equation.

A $y - \frac{1}{2} = 3x$

$y - \frac{1}{2} = 3x$	*Write the original equation.*
$y = 3x + \frac{1}{2}$	*Write the equation in the form y = mx + b.*

The slope of the line is 3 and the y-intercept is $\frac{1}{2}$.

B $y = 4x - 1$

$y = 4x - 1$	*Write the original equation.*
$y = 4x + (-1)$	*Write the equation in the form y = mx + b.*

The slope of the line is 4 and the y-intercept is −1.

Try These Together

State the slope and the y-intercept of the graph of each equation.

1. $y = -x + 2$ **2.** $y = 2x - 6$ **3.** $y = 4x - 1$

PRACTICE

State the slope and the y-intercept of the graph of each equation.

4. $y = \frac{1}{3}x - 12$ **5.** $y = -\frac{1}{2}x + 7$ **6.** $y = -1\backslash 5x - \frac{1}{5}$

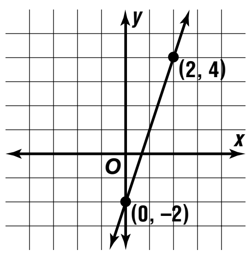
7. $y - x = 4$ **8.** $y + 3x = -1$ **9.** $4x + y = 3$

Graph each equation using the slope and the y-intercept.

10. $y = \frac{1}{4}x - 3$ **11.** $y = \frac{6}{5}x + 2$ **12.** $y = -x - 5$

13. $y = -6x + 2.5$ **14.** $3x + y = 1$ **15.** $y + x = -1$

16. **Standardized Test Practice** What is the equation of the graph at the right?



A $y = \frac{1}{3}x - 2$ **B** $y = -3x + 2$

C $y = -\frac{1}{3}x + 2$ **D** $y = 3x - 2$

Answers: 1. −1; 2 **2.** 2; −6 **3.** 4; −1 **4.** $\frac{1}{3}$; −12 **5.** $-\frac{1}{2}$; 7 **6.** $-\frac{1}{5}$; $-\frac{1}{5}$ **7.** 1; 4 **8.** −3; −1 **9.** −4; 3
10–15. See Answer Key. **16.** D

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11-6 Scatter Plots (pages 539–542)

A graph of two sets of data as ordered pairs is a **scatter plot**. Scatter plots can suggest whether two sets of data are related.

Determining the Relationship	To determine whether two sets of data are related, imagine a line drawn so that half of the points are above the line and half are below it. • A line that slopes upward to the right shows a *positive* relationship. • A line that slopes downward to the right shows a *negative* relationship. • When the points are very spread out instead of clustering along a line, the scatter plot shows that there is *no* relationship between the data sets.

EXAMPLE

Determine whether a scatter plot of the data for age and weight of people younger than 21 would show a *positive, negative,* or *no* relationship.

In children and young people, as the age increases, so does the weight in most cases. A scatter plot of this data would show a positive relationship.

Try These Together

1. Determine whether a scatter plot of the data for bank balance and money spent would show a *positive, negative,* or *no* relationship. Assume everyone considered has the same income.
HINT: Does the bank balance rise or fall as money spent increases?

2. Determine whether a scatter plot of the data for hours of sleep per night and height would show a *positive, negative,* or *no* relationship.
HINT: Do hours of sleep per night and height have any influence on each other?

PRACTICE

Determine whether a scatter plot of the data for the following might show a* positive, negative, *or* no *relationship.

3. temperature and hours of sunlight

4. age past 70 and number of health problems

5. age of a computer and its value

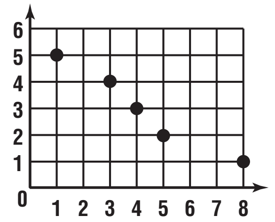
6. hours of battery use and remaining battery life

7. number of seats in a car and the last digit in its license plate number

8. **Standardized Test Practice** What kind of relationship does the scatter plot at the right show?

A positive **B** negative

C no **D** inverse



Answers: **1.** negative **2.** no relationship **3.** positive **4.** positive **5.** negative **6.** negative **7.** no relationship **8.** B

11-7 Graphing Systems of Equations (pages 544–547)

A set of two or more equations is called a **system of equations**. When you find an ordered pair that is a solution of all of the equations in the system, you have solved the system.

Solving Systems of Two Equations by Graphing	The ordered pair that names the point where the two lines intersect (or cross each other) is the solution of the system of equations. The coordinates of this ordered pair make the equations of each of the lines true. Check your solution in both equations.

EXAMPLE

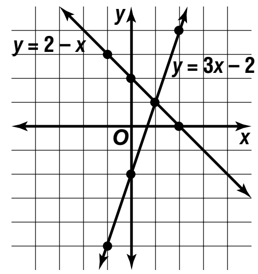
Solve this system of equations by graphing.
$y = 3x - 2$ and $y = 2 - x$

First make a function table for each equation.

x	$3x - 2$	y	(x, y)
−1	3(−1) − 2	−5	(−1, −5)
0	3(0) − 2	−2	(0, −2)
1	3(1) − 2	1	(1, 1)
2	3(2) − 2	4	(2, 4)

x	$2 - x$	y	(x, y)
−1	2 − (−1)	3	(−1, 3)
0	2 − 0	2	(0, 2)
1	2 − 1	1	(1, 1)
2	2 − 2	0	(2, 0)

Graph the ordered pairs for each table and draw each line.



Find the coordinates of the point where the lines cross by looking at the graph. (1, 1)
Check this solution in both equations.
Does 1 = 3(1) −2? yes
Does 1 = 2 −1? yes
The solution of this system is (1, 1).

Try These Together

1. Solve the system $y = 2x + 3$ and $y = x + 1$ by graphing.
HINT: The lines intersect in Quadrant III.

2. Solve the system $y = x + 2$ and $y = 2x + 2$ by graphing.
HINT: Choose at least 3 values for x in each equation.

PRACTICE

Solve each system of equations by graphing.

3. $y = 4x + 4$
$y = 3x + 2$

4. $x + y = 9$
$y = 13 - 2x$

5. $2 - x = y$
$3x + 14 = y$

6. **Standardized Test Practice** You are walking along the path of $y = 6x + 8$ and your friend Ramon is walking on the path of $y = 8x + 12$. At what point do your paths cross?

A (0, 8) **B** (−1, 4) **C** (−2, −4) **D** (1, 14)

Answers: 1–5. See Answer Key for graphs. **1.** (−2, −1) **2.** (0, 2) **3.** (−2, −4) **4.** (4, 5) **5.** (−3, 5) **6.** C

11-8 Graphing Linear Inequalities (pages 548–551)

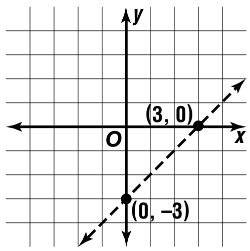
Graphing Linear Inequalities	To graph an inequality, first graph the related equation. This is the boundary. If the inequality contains the symbol $\leq$ or $\geq$, a solid line is used to indicate that the boundary is included in the graph. If the inequality contains the symbol $<$ or $>$, a dashed line is used to indicate that the boundary is not included in the graph.
	Next, test any point above or below the line to determine which region is the solution of the inequality.

EXAMPLE

Graph $y > x - 3$.

Graph the boundary line $y = x - 3$.

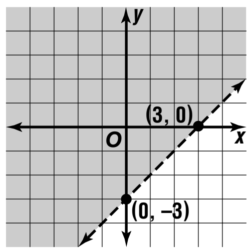
Since $>$ is used in the inequality, make the boundary line dashed.



Test a point not on the boundary line, such as (0, 0).

$y > x - 3$	*Write the inequality.*
$0 \overset{?}{>} 0 - 3$	*Replace x with 0 and y with 0.*
$0 > -3$	*Simplify.*

Since (0, 0) is a solution of $y > x - 3$, shade the region that contains (0, 0).



Try These Together

Graph each inequality.

1. $y \geq 3x + 2$ **2.** $y < \frac{1}{2}x - 1$ **3.** $y > \frac{3}{2}x + 3$

PRACTICE

Graph each inequality.

4. $y < x + 6$ **5.** $y \geq 3x - 7$ **6.** $y > -2x - 2$

7. $y = -\frac{1}{2}x - 2$ **8.** $y < -x + 1$ **9.** $y \geq \frac{4}{3}x + 2$

10. $y > 6x - 1$ **11.** $y - x \leq -3$ **12.** $y + 3x < 5$

13. **Standardized Test Practice** Which ordered pair is *not* a solution of $y \leq \frac{1}{2}x + 1$?

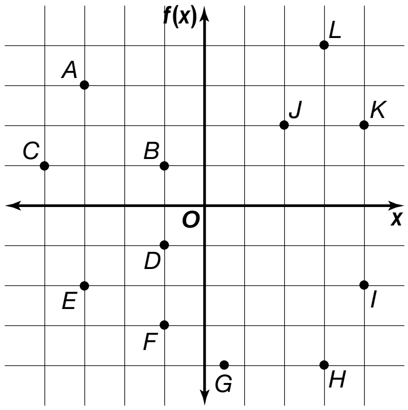
A (0, 0) **B** (2, 3) **C** (3, 1) **D** (4, 1)

Answers: 1–12. See Answer Key. **13.** B

11 Chapter 11 Review

Function Map

Franny's friends leave her a map so she can find their picnic in the park. The picnic site is located somewhere on the graph of the function $f(x) = -2x - 3$.



1. Complete the function table for $f(x) = -2x - 3$

x	$-2x - 3$	$f(x)$
0	$-2(0) - 3$	-3
-1		
-2		

2. Graph the function on the map above.

3. Which points on the map could possibly be the picnic site?

4. If the picnic site is in Quadrant II on the map, which point is it?

5. There is a swing set that is also on the graph of the function in Quadrant III. Which point is the swing set?

6. A large pecan tree is on the graph of the function in Quadrant IV. Which point is the pecan tree?

Answers are located on page 113.