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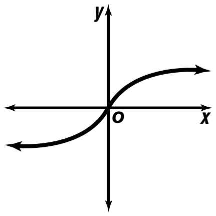
12-1 Linear and Nonlinear Functions (pages 560–563)

Linear functions have graphs that are straight lines. These graphs represent constant rates of change. Nonlinear functions do not have constant rates of change. Therefore, their graphs are not straight lines.

EXAMPLES

Identify Functions Using Graphs



The graph is a curve, not a straight line. So it represents a nonlinear function.

Identify Functions Using Equations

$y = x^2 - 1$
Since x is raised to a power, the equation cannot be written in the form $y = mx + b$. So this function is nonlinear.

Identify Functions Using Tables

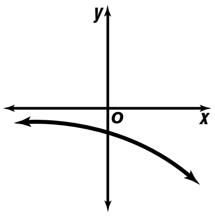
x	5	7	9	11
y	8	12	16	20

As x increases by 2, y increases by 4 each time.
The rate of change is constant, so this function is linear.

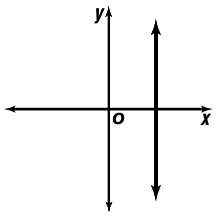
PRACTICE

Determine whether each graph, equation, or table represents a* linear *or* nonlinear *function. Explain.

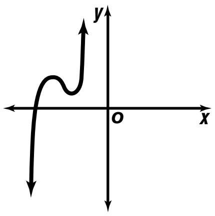
1.



2.



3.



4. $y = -2$

5. $y = x^2$

6. $x - y = 5$

7.

x	3	4	5	6
y	10	11	12	13

8.

x	3	6	9	12
y	−4	−1	3	8

9.

x	−3	−2	−1	0
y	4	9	16	25

10. Standardized Test Practice Which equation represents a linear function?

A $x + y = 4$ **B** $y = \frac{6}{x}$ **C** $xy = 3$ **D** $y = x^3 - 1$

Answers: 1. nonlinear; graph is a curve **2.** linear; graph is a straight line **3.** nonlinear; graph is a curve **4.** linear; can be written as $y = 0x + -2$ **5.** nonlinear; power of x is greater than one **6.** linear; can be written as $y = x + -5$ **7.** linear; rate of change is constant, as x increases by 1, y increases by 1 **8.** nonlinear; rate of change is not constant, as x increases by 3, y increases by a greater amount each time **9.** nonlinear; rate of change is not constant, as x increases by 1, y increases by a greater amount each time **10.** A

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12-2 Graphing Quadratic Functions (pages 565–568)

In a quadratic function, the greatest power of the input variable (usually x) is 2. For example, $y = x^2$, $A = s^2$, and $y = 3x^2 + 5$ are all quadratic functions.

Graphing Quadratic Functions	You graph a quadratic function with the same steps you used for graphing a linear function, but the graph of a quadratic function is a curve, not a straight line. The graphs of the quadratic functions in this lesson are all curves, called **parabolas**, shaped a little like the letter U.

EXAMPLE

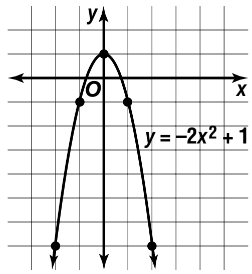
Graph the quadratic function $y = -2x^2 + 1$.

Choose some values for x and make a table.

x	$-2x^2 + 1$	y	(x, y)
-2	$-2(-2)^2 + 1 = -7$	-7	$(-2, -7)$
-1	$-2(-1)^2 + 1 = -1$	-1	$(-1, -1)$
0	$-2(0)^2 + 1 = 1$	1	$(0, 1)$
1	$-2(1)^2 + 1 = -1$	-1	$(1, -1)$
2	$-2(2)^2 + 1 = -7$	-7	$(2, -7)$

Because the graph is a curve, plot more points than you would for a straight line, so that you can see the shape of the curve.

Graph the (x, y) points in the last column of your table. Draw a smooth curve to join the points.



Try These Together

1. Complete the function table and then graph the function $y = 2x^2$.

x	$2x^2$	y	(x, y)
-2			
-1			
0			
1			
2			

HINT: The y-values repeat.

2. Complete the function table and then graph the function $f(x) = \frac{1}{2}x^2$.

x	$\frac{1}{2}x^2$	$f(x)$	$(x, f(x))$
-4			
-2			
0			
2			
4			

HINT: Treat the f(x) like y.

PRACTICE

3. Graph $f(x) = 2x^2 - 5$.

4. Graph $y = 12 - x^2$.

5. **Standardized Test Practice** Determine which ordered pair is a solution of $y = x^2 + x - 3$.

A (6, 9) **B** (2, −1) **C** (4, 17) **D** (−3, −15)

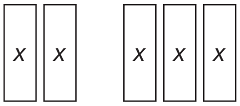
Answers: 1–4. See Answer Key for graphs. **1.** (−2, 8), (−1, 2), (0, 0), (1, 2), (2, 8) **2.** (−4, 8), (−2, 2), (0, 0), (2, 2), (4, 8) **5.** C

12-3 Simplifying Polynomials (pages 570–573)

Each monomial in a polynomial is called a **term**. Monomials with the same variable to the same power, such as $2x$ and $3x$, are called **like terms**. You can simplify polynomials that have like terms. An expression that has no like terms is in **simplest form**.

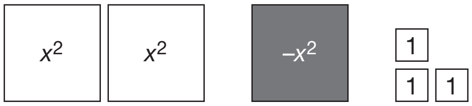
EXAMPLES

A Simplify $2x + 3x$.



With tiles you can see that there are 5 x-tiles. On paper, you add the like terms. So $2x + 3x = 5x$.

B Simplify $2x^2 - x^2 + 3$.



With the tiles, you can see that there are 2 positive x^2-tiles and one negative x^2-tile. Two positives plus one negative equals one positive. Or, on paper, $2x^2 - x^2 = x^2$. So the polynomial in simplest form is $x^2 + 3$.

Try These Together

***Simplify each polynomial. If the polynomial cannot be simplified, write* simplest form.**

1. $3 + 2q^2 - 3 + q^2$ **2.** $4r^2 - 2r^2 + r$ **3.** $3z + 2y - 5x + 2$

HINT: Monomials with the same variable and power are like terms. All numbers without variables are like terms.

PRACTICE

***Simplify each polynomial. If the polynomial cannot be simplified, write* simplest form.**

4. $5a^2 - 2a + 3$ **5.** $6d + 2r - 3d$ **6.** $c^2 - 4c + 3$

7. $m^4 + m - m^2 + m$ **8.** $-1 + x^4 - x^2 + x + 5$ **9.** $t^3 + t^3 - t^3$

10. $y^3 - y^3 + y^2 + 3y^3$ **11.** $w^2 + 4w - 1$ **12.** $5g - 2h + g - 3h$

13. $2b + 3 + 4b + 2$ **14.** $x^2 + 2x + 3x^2 + 4$ **15.** $2r^2 + 4r + 3r + r^2 + r$

16. $a - b + 3b - 1$ **17.** $2y + 2y^2 - 2y^2 + y$ **18.** $3a^3 + 2a^2 + a$

19. Money Matters César put his \$50 cash birthday gift in a savings account. He also received \$50 last year and also put it in the account. Adding the interest x he made from his account, write an expression in simplest form that represents the amount of money in his account.

20. Standardized Test Practice Simplify the polynomial $x^2 + x + 2x^2 + 3$.

A $x^2 + 2x + 3$ **B** $4x^2 + 2x + 3$ **C** $3x^2 + x + 3$ **D** $2x^2 + x + 3$

Answers: 1. $3q^2$ **2.** $2r^2 + r$ **3.** simplest form **4.** simplest form **5.** $3d + 2r$ **6.** simplest form **7.** $m^4 - m^2 + 2m$ **8.** $x^4 - x^2 + x + 4$ **9.** t^3 **10.** $3y^3 + y^2$ **11.** simplest form **12.** $6g - 5h$ **13.** $6b + 5$ **14.** $4x^2 + 2x + 4$ **15.** $3r^2 + 8r$ **16.** $a + 2b - 1$ **17.** $3y$ **18.** simplest form **19.** $100 + x$ **20.** C

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12-4 Adding Polynomials (pages 574–577)

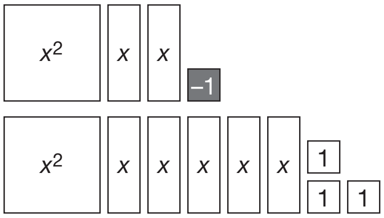
To add polynomials, add the like terms in each polynomial. You can use algebra tiles or pencil and paper to add polynomials.

EXAMPLES

Find each sum.

A $(x^2 + 2x - 1) + (x^2 + 5x + 3)$

Use algebra tiles to represent each polynomial.



Using the tiles, add like terms to find the sum, $2x^2 + 7x + 2$.

B $(2x^2 - x + 2) + (-x^2 + 3x + 2)$

Align the like terms in columns, then add.

$$\begin{array}{r} 2x^2 - x + 2 \\ + (-x^2) + 3x + 2 \\ \hline x^2 + 2x + 4 \end{array}$$

Try These Together

Add.

1. $\begin{array}{r} y^2 + 2y + 1 \\ + y^2 - 3y - 2 \\ \hline \end{array}$

HINT: $2y + (-3y) = -y$

2. $\begin{array}{r} 3x^2 + y + 3 \\ + 2x^2 - 3y + 4 \\ \hline \end{array}$

HINT: $y + (-3y) = -2y$

3. $\begin{array}{r} 4m^2 + 2m + 5 \\ + 3m^2 + m - 4 \\ \hline \end{array}$

HINT: Like terms are in columns.

PRACTICE

Add.

4. $\begin{array}{r} 7x^2 - 6x - 2 \\ + 5x^2 + 3x - 4 \\ \hline \end{array}$

5. $\begin{array}{r} 10q^2 + 7q + 1 \\ + 8q^2 + 2q - 6 \\ \hline \end{array}$

6. $\begin{array}{r} 4a^2 + 4a + 4 \\ + (-3a^2) - 3a - 3 \\ \hline \end{array}$

Add. Then evaluate each sum if x = 3 and y = 2.

7. $(3x + 2y) + (2 + 3y)$

8. $(4x + y) + (-2x + 2y)$

9. $(-2x + 3y) + (3x - 4y)$

10. $(-4x - 3y) + (-x - y)$

11. $(5x + 3y) + (4x + 3y)$

12. $(x + y) + (y + x)$

13. **Standardized Test Practice** What is the sum of $t^2 + 2t + 1$ and $t^2 + 3t + 2$?

A $t^2 + t + 3$ **B** $2t^2 + 5t + 3$ **C** $2t^2 + 5t^2 + 3$ **D** $t^2 + 5t + 3$

Answers: 1. $2y^2 - y - 1$ **2.** $5x^2 - 2y + 7$ **3.** $7m^2 + 3m + 1$ **4.** $12x^2 - 3x - 6$ **5.** $18q^2 + 9q - 5$ **6.** $a^2 + a + 1$ **7.** $3x + 5y + 2$; 21 **8.** $2x + 3y$; 12 **9.** $x - y$; 1 **10.** $-5x - 4y$; −23 **11.** $9x + 6y$; 39 **12.** $2x + 2y$; 10 **13.** B

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12-5 Subtracting Polynomials (pages 580–583)

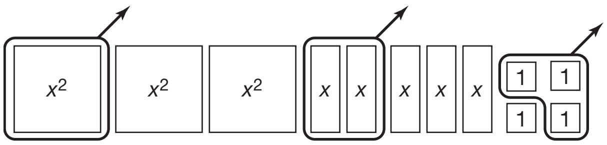
Subtracting polynomials is very similar to adding polynomials. You can use algebra tiles to subtract polynomials. You can also use paper and pencil. Since subtracting is the same as adding the opposite, use this procedure to subtract polynomials with paper and pencil.

EXAMPLES

Find each difference.

A $(3x^2 + 5x + 4) - (x^2 + 2x + 3)$

Use algebra tiles to represent the first polynomial.



To subtract, remove the tiles representing the second polynomial. The remaining tiles represent the difference, $2x^2 + 3x + 1$.

B $(2x^2 + 4x + 3) - (-x^2 + 3x + 2)$

Subtracting $-x^2 + 3x + 2$ is the same as adding the additive inverse. To find the additive inverse, find the opposite of the term, or $x^2 - 3x - 2$.

$$\begin{array}{r} 2x^2 + 4x + 3 \\ + \; x^2 - 3x - 2 \\ \hline 3x^2 + \;\; x + 1 \end{array}$$

Try These Together

Subtract.

1. $\begin{array}{r} 4x + 4 \\ -\,(2x + 6) \\ \hline \end{array}$

HINT: The additive inverse of $2x + 6$ is $-2x - 6$.

2. $\begin{array}{r} 3x + 5 \\ -\,(x - 1) \\ \hline \end{array}$

HINT: The additive inverse of $x - 1$ is $-x + 1$.

3. $\begin{array}{r} 10x + 5 \\ -\,(5x + 1) \\ \hline \end{array}$

HINT: Add the additive inverse.

PRACTICE

Subtract.

4. $\begin{array}{r} 7y + 2 \\ -\,(4y + 3) \\ \hline \end{array}$

5. $\begin{array}{r} 8r^2 + 5a + 5 \\ -\,(6r^2 + 3a + 2) \\ \hline \end{array}$

6. $\begin{array}{r} 7a^2 + 4a + 4 \\ -\,(5a^2 + 2a + 2) \\ \hline \end{array}$

7. $(4b^2 + 4b + 4) - (-b^2 + b - 1)$

8. $(3b^2 + 3b + 3) - (2b^2 - 2b + 2)$

Subtract. Then evaluate if $x = -3$ and $y = 4$.

9. $(6x + 3y) - (3x + 2y)$

10. $(5x + 5y) - (4x + 4y)$

11. **Standardized Test Practice** Subtract $(5x + 3y) - (2x + 4y)$ then evaluate if $x = -2$ and $y = 5$.

A 13 **B** -29 **C** 6 **D** -11

Answers: **1.** $2x - 2$ **2.** $2x + 6$ **3.** $5x + 4$ **4.** $3y - 1$ **5.** $2r^2 + 2a + 3$ **6.** $2a^2 + 2a + 2$ **7.** $5b^2 + 3b + 5$ **8.** $b^2 + 5b + 1$ **9.** $3x + y$; -5 **10.** $x + y$; 1 **11.** D

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12-6 Multiplying and Dividing Monomials

(pages 584–587)

In order to multiply and divide monomials, you will multiply and divide powers that have the same base.

Product of Powers	You can multiply powers that have the same base by adding their exponents. So, for any number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.
Quotient of Powers	You can divide powers that have the same base by subtracting their exponents So, for any number a and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

EXAMPLES

Multiply or divide. Express using exponents.

A $x^3 \cdot x^5$

$x^3 \cdot x^5 = x^{3+5}$ *or* x^8

B $\frac{d^6}{d^2}$

$\frac{d^6}{d^2} = d^{6-2}$ *or* d^4

Try These Together

Multiply or divide. Express using exponents.

1. $b \cdot b^4$ **2.** $\frac{x^5}{x^3}$ **3.** $3^2 \cdot 3^2$

HINT: When you multiply powers, use the same base and use a new exponent that is the sum of the original ones. When you divide powers, the new exponent is the difference of the original ones. Bases with no exponent written have an understood exponent of 1.

PRACTICE

Multiply. Express using exponents.

4. $r^3 \cdot r^3$ **5.** $2r^2 \cdot r^2$ **6.** $3a \cdot a^5$

7. $2c \cdot c^4$ **8.** $x^5 \cdot x^{10}$ **9.** $4^7 \cdot 4^9$

Divide. Express using exponents.

10. $\frac{b^{12}}{b^7}$ **11.** $\frac{8m^7}{2m^3}$ **12.** $\frac{9^8}{9^2}$

13. $\frac{12y^5}{3y^4}$ **14.** $\frac{6^4}{6}$ **15.** $\frac{f^{14}}{f^9}$

16. **Standardized Test Practice** Find the product $2x^6 \cdot x^{10}$.

A $2x^{16}$ **B** x^{16} **C** $2x^4$ **D** $2x^{60}$

Answers: **1.** b^5 **2.** x^2 **3.** 3^4 **4.** r^6 **5.** $2r^4$ **6.** $3a^6$ **7.** $2c^5$ **8.** x^{15} **9.** 4^{16} **10.** b^5 **11.** $4m^4$ **12.** 9^6 **13.** $4y$ **14.** 6^3 **15.** f^5 **16.** A

12-7 Multiplying Monomials and Polynomials

(pages 590–592)

You can multiply monomials and polynomials by using the Distributive Property. Often, the definition of exponents and the Product of Powers rule are also needed to simplify the product of a monomial and a polynomial.

EXAMPLES

A Find $2b(b + 6)$.

$2b(b + 6) = 2b(b) + 2b(6)$ *Distributive Property*

$= 2b^2 + 12b$ $\quad b \cdot b = b^2$

B Find $g^3(g - 2)$.

$g^3(g - 2) = g^3[g + (-2)]$ *Rewrite $g - 2$ as $g + (-2)$.*

$= g^3(g) + g^3(-2)$ *Distributive Property*

$= g^4 + (-2g^3)$ $\quad g^3(g) = g^{3+1}$ *or* g^4

$= g^4 - 2g^3$ *Definition of subtraction*

Try These Together

Multiply.

1. $-4y(y + 2)$

2. $n(3n^2 - n + 8)$

HINT: Use the Distributive Property, and add exponents when multiplying powers with the same base.

PRACTICE

Multiply.

3. $(x + 2)(4x)$

4. $a^3(a - 3)$

5. $y^4(y^4 + 6)$

6. $5m^3(m^2 + 1)$

7. $y(y^2 + 4y - 3)$

8. $-x^2(x^3 + 2)$

9. $2q^2(2q - 1)$

10. $-a(a + 4)$

11. $n(3n^2 - 4n + 7)$

12. $r^3(r^5 - r^3 - 5)$

13. $(w^2 + 6)(5w)$

14. $3q^2(q^2 + 2)$

15. **Standardized Test Practice** What is the product of $2z^2$ and $4z^2 + 2z - 8$?

A $8z^4 + 4z^2 + 2z - 8$

B $8z^4 + 4z^3 - 16z^2$

C $8z^2 + 4z - 16$

D $8z^4 + 4z^3 + 2z^2 - 16$

Answers: **1.** $-4y^2 - 8y$ **2.** $3n^3 - n^2 + 8n$ **3.** $4x^2 + 8x$ **4.** $a^4 - 3a^3$ **5.** $y^8 + 6y^4$ **6.** $5m^5 + 5m^3$ **7.** $y^3 + 4y^2 - 3y$ **8.** $-x^5 - 2x^2$ **9.** $4q^3 - 2q^2$ **10.** $-a^2 - 4a$ **11.** $3n^3 - 4n^2 + 7n$ **12.** $r^8 - r^6 - 5r^3$ **13.** $5w^3 + 30w$ **14.** $3q^4 + 6q^2$ **15.** B

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12 Chapter 12 Review

Match'Em

First, simplify the expressions in each column. Each expression in the left column matches exactly one expression in the right column. Write the correct letter in the blank next to each expression in the left column.

________	**1.** $2x - 1 + 2x + 2$	**A.** $\frac{45x^9}{3x^2}$
________	**2.** $(4x)^2$	**B.** $6x^4 (4x)$
________	**3.** $6x(x + 2)$	**C.** $4(4x^2)$
________	**4.** $\frac{4^{12}}{4^3}$	**D.** $x(-3x^2 + 6x - 12)$
________	**5.** $(2x^2 + x + 1) - (3x^2 - x - 1)$	**E.** $(7x^2 + x) - (4x + 1)$
________	**6.** $-3x^5 (-5x^2)$	**F.** $2(x - 5)$
________	**7.** $6x^3 \cdot 6x^4$	**G.** $3x^2 + 4x + 5x - 3 - 2x^2$
________	**8.** $(7x^2 - 3x) + (2x^2 + 2x)$	**H.** $(-5x^2 + 2x - 1) + (4x^2 + 3)$
________	**9.** $-3x(x^2 - 2x + 4)$	**I.** 2
________	**10.** $6x^2 - 3x + x^2 - 1$	**J.** $9x(4x^6)$
________	**11.** $9x + x^2 - 3$	**K.** $3x + x + 1$
________	**12.** $\frac{-20x^3}{4x^2}$	**L.** $-x(5x^2)$
________	**13.** $-8x^3(-3x^2)$	**M.** $(4^3)^3$
________	**14.** $\frac{6x^3}{3x^4}$	**N.** $5x^2 + 13x - x + x^2$
________	**15.** $(13x^2 - 2x - 10) + (-13x^2 + 4x)$	**O.** $(4x^2 - 5x) - (-5x^2 - 4x)$

Answers are located on page 114.