1 The Tangent Ratio

***What** you'll learn*

You'll learn to find the tangent of an angle and find missing measures using the tangent.

***When** am I ever going to use this?*

Knowing how to use the tangent ratio can help you find unknown measures in triangles.

Word Wise

tangent

The industrial technology class plans to add a wheelchair ramp to the emergency exit of the auditorium as a class project. They know that the landing is 3 feet high and that the angle the ramp makes with the ground cannot be greater than 6°. What is the minimum distance from the landing that the ramp should start? *This problem will be solved in Example 1.*

Problems like the one above involve a right triangle and ratios. One ratio, called the **tangent**, compares the measure of the leg opposite an angle with the measure of the leg adjacent to that angle. The symbol for the tangent of angle A is $\tan A$.

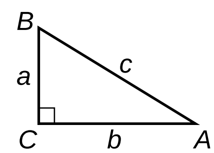
Tangent Ratio

Words: If A is an acute angle of a right triangle,

$$\tan A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the leg adjacent to } \angle A}.$$

Symbols: $\tan A = \frac{a}{b}$

Model:



You can also use the symbol for tangent to write the tangent of an angle measure. The tangent of a 60° angle is written as tan 60°. If you know the measures of one leg and an acute angle of a right triangle, you can use the tangent ratio to solve for the measure of the other leg.

Example 1 APPLICATION

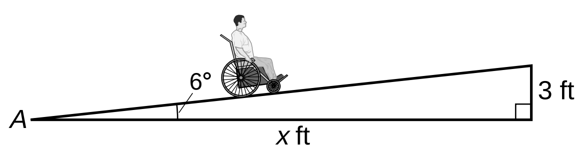
Construction **Solve the problem about the wheelchair ramp.**

First, draw a diagram.

$m\angle A = 6°$

adjacent leg = x feet

opposite leg = 3 feet



Now substitute these values into the definition of tangent.

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

$$\tan 6° = \frac{3}{x}$$

$$(\tan 6°)(x) = 3 \quad \textit{Multiply each side by } x.$$

$$x = \frac{3}{\tan 6°} \quad \textit{Divide each side by } \tan 6°.$$

3 [÷] 6 [TAN] [=] 28.54309336

To the nearest tenth, the ramp should begin about 28.5 feet from the landing.

If your calculator does not have a [TAN] *key, you can use the table on the back cover of this booklet to estimate answers.*

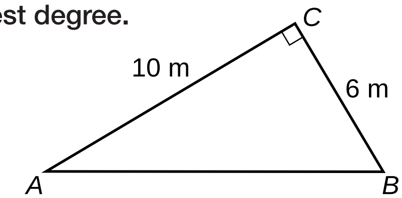
You can use the TAN^{-1} function on your calculator to find the measure of an acute angle of a right triangle when you know the measures of the two legs.

Example

Study Hint

Technology To find TAN^{-1}, press the [2nd] key and then the [TAN] key.

2 **Find the measure of ∠*A* to the nearest degree.**



From the figure, you know the measures of the two legs. Use the definition of tangent.

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

$\tan A = \frac{6}{10}$

Now use your calculator to find the measure of ∠*A*.

6 [÷] 10 [=] [2nd] [TAN^{-1}] 30.96375653

The measure of *A* is about 31°.

CHECK FOR UNDERSTANDING

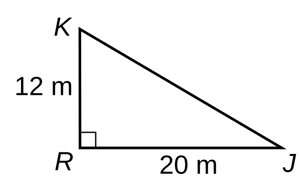
Communicating Mathematics

Read and study the lesson to answer each question.

1. ***Write*** a definition of the tangent ratio.
2. ***Tell*** how to use the tangent ratio to find the measure of a leg of a right triangle.
3. ***Tell*** how to find the measure of an angle in a right triangle when you know the measures of the two legs.

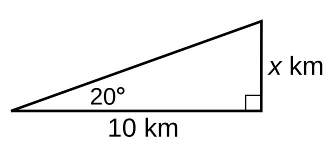
Guided Practice

Find each tangent to the nearest tenth. Find the measure of each angle to the nearest degree.



4. tan *J*
5. tan *K*
6. *m*∠*J*
7. *m*∠*K*

8. Find the value of *x* to the nearest tenth.

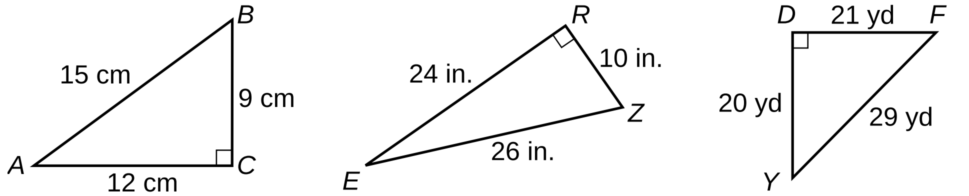


9. ***Measurement*** A guyline is fastened to a TV tower 50 feet above the ground and forms an angle of 65° with the tower. How far is it from the base of the tower to the point where the guyline is anchored into the ground? Round to the nearest foot.

EXERCISES

Practice

Complete each exercise using the information in the figures. Find each tangent to the nearest tenth. Find the measure of each angle to the nearest degree.



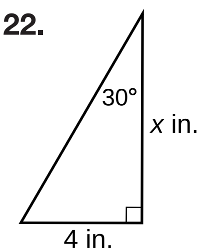
10. $\tan A$ **11.** $\tan B$ **12.** $m\angle A$ **13.** $m\angle B$

14. $\tan Z$ **15.** $\tan E$ **16.** $m\angle Z$ **17.** $m\angle E$

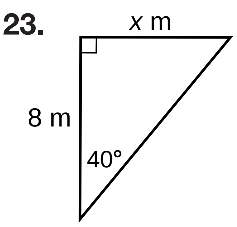
18. $\tan F$ **19.** $\tan Y$ **20.** $m\angle F$ **21.** $m\angle Y$

Find the value of *x* to the nearest tenth.

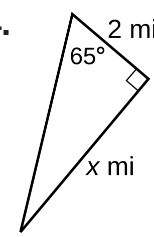
22.



23.



24.

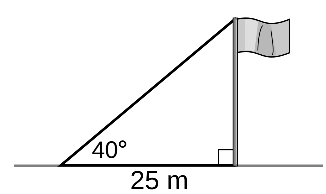


25. If the leg opposite the 53° angle in a right triangle is 4 inches long, how long is the other leg to the nearest tenth?

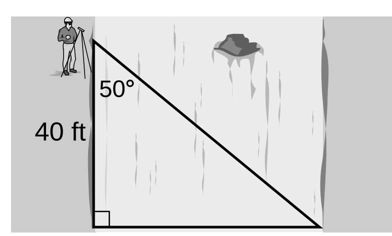
26. If the leg adjacent to a 29° angle in a right triangle is 9 feet long, what is the measure of the other leg to the nearest tenth?

Applications and Problem Solving

27. ***Measurement*** A flagpole casts a shadow 25 meters long when the angle of elevation of the Sun is 40°. How tall is the flagpole to the nearest meter?



28. ***Surveying*** A surveyor is finding the width of a river for a proposed bridge. A theodolite is used by the surveyor to measure angles. The distance from the surveyor to the proposed bridge site is 40 feet. The surveyor measures a 50° angle to the bridge site across the river. Find the length of the bridge to the nearest foot.



29. They are equal.

29. ***Critical Thinking*** In a right triangle, the tangent of one of the acute angles is 1. Describe how the measures of the two legs are related.

Name ______________________________ Date ______________

Study Guide

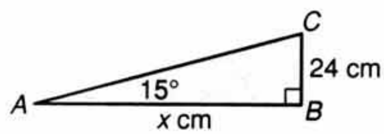
The Tangent Ratio

The tangent ratio compares the measure of the leg opposite an angle with the measure of the leg adjacent to that angle.

If A is an acute angle of a right triangle, then

$$\tan A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the leg adjacent to } \angle A}$$

Example 1 Use the tangent ratio to find the value of x.
From the diagram: $m\angle A = 15°$
adjacent leg = x cm
opposite leg = 24 cm



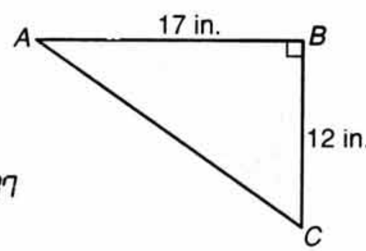
$\tan 15° = \frac{24}{x}$ ← opposite leg / ← adjacent leg

$(\tan 15°)(x) = 24$

$x = \frac{24}{\tan 15°}$

Use a calculator: 24 [÷] 15 [TAN] [=] 89.56921938
$x \approx 89.6$

Example 2 Find the measure of $\angle A$.



$\tan A = \frac{12}{17}$ ← opposite leg / ← adjacent leg
Use a calculator:
12 [÷] 17 [=] [2nd] [TAN⁻¹] 35.21759297
$m\angle A \approx 35°$

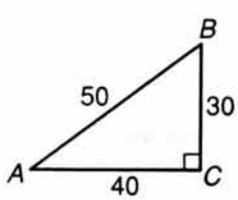
Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

1. $\tan A$ = _____

$\tan B$ _____

$m\angle A$ = _____

$m\angle B$ = _____

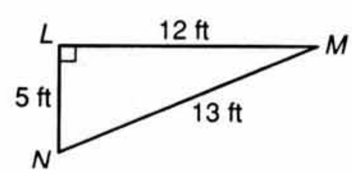


2. $\tan M$ = _____

$\tan N$ = _____

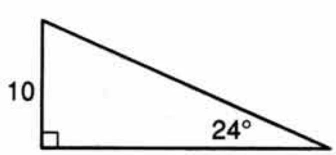
$m\angle M$ = _____

$m\angle N$ = _____



Find the value of x to the nearest tenth or degree.

3.



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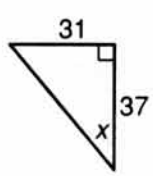

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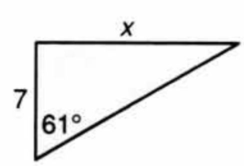


Name ______________________ Date __________

1 Practice

The Tangent Ratio

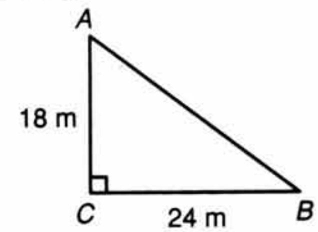
Use the figures at the right for Exercises 1-8. Write the ratios in simplest form. Find angle measures to the nearest degree.

1. Find tan A.

2. Find $m\angle A$.

3. Find tan B.

4. Find $m\angle B$.

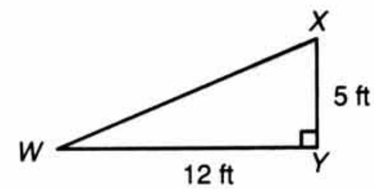


5. Find tan W.

6. Find $m\angle W$.

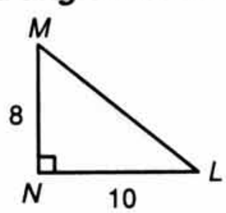
7. Find tan X.

8. Find $m\angle X$.



Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

9.



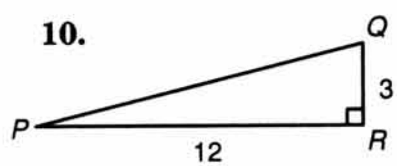
tan L = ?

$m\angle L$ = ?

tan M = ?

$m\angle M$ = ?

10.



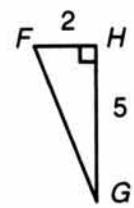
tan P = ?

$m\angle P$ = ?

tan Q = ?

$m\angle Q$ = ?

11.



tan F = ?

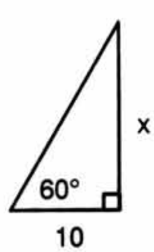
$m\angle F$ = ?

tan G = ?

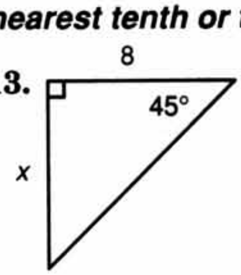
$m\angle G$ = ?

Find the value of x to the nearest tenth or to the nearest degree.

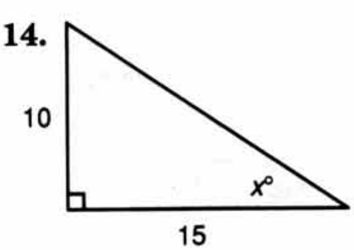
12.



13.



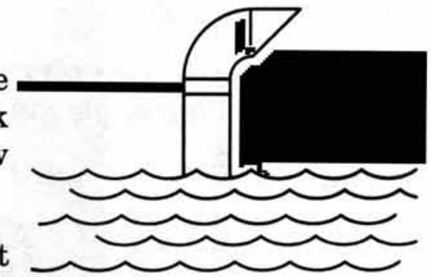
14.



Name ______________________ Date ________

Enrichment

On a Clear Day



If you've ever watched a submarine movie, you've probably seen the captain of the submarine look through a periscope. Have you ever wondered how far the captain could see?

The equation that approximates the distance that someone can see is $d = 1.5\sqrt{h}$ where d is the distance in miles and h is the height in feet of the observer above the surface.
When using this formula, remember that it gives answers only for ideal conditions. Viewing conditions on Earth are typically less than ideal.

Use the equation $d = 1.5\sqrt{h}$ and your calculator to answer the following questions. When necessary, round your answers to the nearest hundredth.

1. A submarine captain is looking through a periscope that is 1 foot above the water. How far is the captain able to see?

2. An airplane is flying at an altitude of 6.5 miles. When looking out the window, how far would an observer be able to see? (5,280 feet =1 mile)

3. The Sears Tower in Chicago is 1,464 feet tall. If the observation deck is located at a height of 1,350 feet above ground, how far would a person on the observation deck be able to see?

4. A sailor is standing in the crow's nest of a sailing ship. The crow's nest is an observation area located high above the ship on a mast. The sailor in the crow's nest is 120 feet above the surface of the water. How far is the sailor able to see?

5. Using a periscope, a submarine captain sights a rowboat that is floating at a distance of 1.84 miles from the ship. How far is the periscope above water?

6. When looking out the window of an airplane, an observer is able to see a distance of 150 miles. At what height is the airplane flying?

2 The Sine and Cosine Ratios

What you'll learn

You'll learn to find the sine and cosine of an angle and find missing measures using sine and cosine.

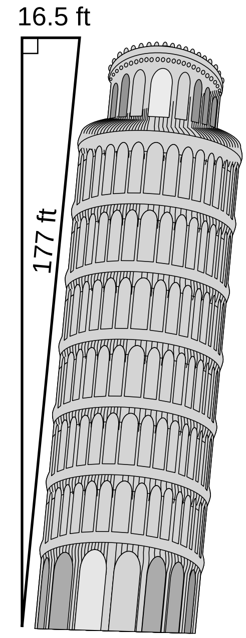
When am I ever going to use this?

Knowing how to use trigonometric ratios can help you solve problems using angles of elevation.

Word Wise

trigonometry
sine
cosine
angle of elevation

Toni decided to make a scale drawing of the Leaning Tower of Pisa for her project in art class. She knows the tower is 177 feet tall and tilts 16.5 feet off the perpendicular. First, she wants to draw the angle representing the tilt of the tower. What should be the measure of this angle? *This problem will be solved in Example 3.*



You know the measures of one leg and the hypotenuse of a right triangle. These are *not* the measures you need to use the tangent ratio. The tangent ratio is only one of several ratios used in the study of **trigonometry.**

Two other ratios are the **sine** ratio and the **cosine** ratio. These can be written as sin A and cos A. They are defined as follows.

Sine and Cosine Ratios

Words: If A is an acute angle of a right triangle,

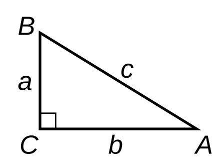
$\sin A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the hypotenuse}}$ and

$\cos A = \frac{\text{measure of the leg adjacent to } \angle A}{\text{measure of the hypotenuse}}$.

Symbols: $\sin A = \frac{a}{c}$

$\cos A = \frac{b}{c}$

Model:



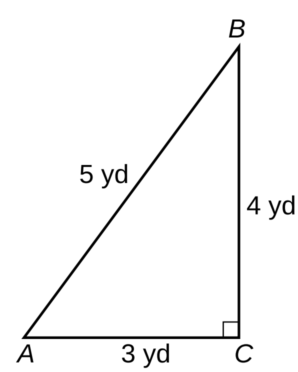
Example 1 Use $\triangle ABC$ to find sin A, cos A, sin B, and cos B.

$\sin A = \frac{BC}{AB}$
$= \frac{4}{5}$ or 0.8

$\sin B = \frac{AC}{AB}$
$= \frac{3}{5}$ or 0.6

$\cos A = \frac{AC}{AB}$
$= \frac{3}{5}$ or 0.6

$\cos B = \frac{BC}{AB}$
$= \frac{4}{5}$ or 0.8



You can find the sine and cosine of an angle by using a calculator.

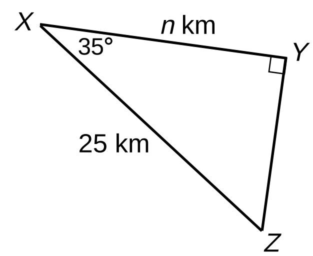
sin 63° → 63 [SIN] 0.891006524

cos 63° → 63 [COS] 0.4539905

You can use the sine and cosine ratios to find missing lengths of sides or angle measures in a right triangle.

Examples

2 **Find the length of $\overline{XY}$ in △*XYZ*.**



You know the measure of ∠*X* and the length of the hypotenuse. You can use the cosine ratio.

$\cos X = \frac{XY}{XZ}$ $\frac{\text{adjacent leg}}{\text{hypotenuse}}$

$\cos 35° = \frac{n}{25}$ *Replace X with 35°, XY with n, and XZ with 25.*

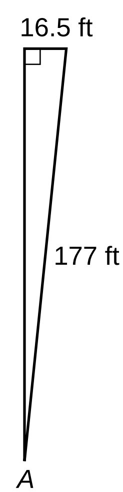
$(25)(\cos 35°) = n$ *Multiply each side by 25.*

25 [×] 35 [COS] [=] 20.47880111

The length of $\overline{XY}$ is about 20.5 kilometers.

APPLICATION

3 **Scale Drawings** **Find the angle that Toni needs to draw for her scale drawing of the Leaning Tower of Pisa.**



Explore You know the length of the leg opposite the angle and the length of the hypotenuse. You can use sin *A*.

Plan Substitute the known values into the definition of sine.

$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$

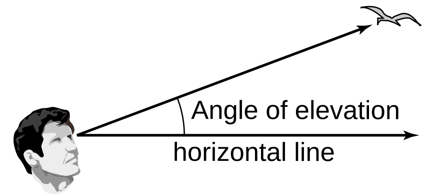
Solve $\sin A = \frac{16.5}{177}$

16.5 [÷] 177 [=] [2nd] [SIN^{-1}]
5.348898164

Toni must draw an angle of about 5°.

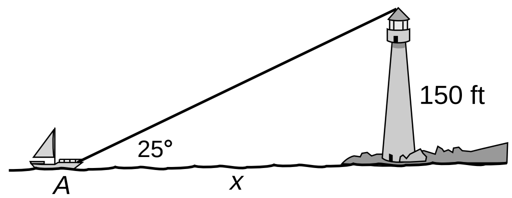
Examine Toni knows the angle in her drawing will be very narrow. Since 5° is a very small angle, it is a reasonable answer.

Many problems that can be solved using trigonometric ratios deal with angles of elevation. An **angle of elevation** is formed by a horizontal line and a line of sight above it.



Example 4 INTEGRATION

Measurement **The angle of elevation from a small boat to the top of a lighthouse is 25°. If the top of the lighthouse is 150 feet above sea level, find the distance from the boat to the base of the lighthouse.**



Let x = the distance from the boat to the base of the lighthouse.

$\tan 25° = \frac{150}{x}$ *opposite leg / adjacent leg*

$(\tan 25°)\, x = 150$ *Multiply each side by x.*

$x = \frac{150}{\tan 25°}$ *Divide each side by tan 25°.*

150 [÷] 25 [TAN] [=] 321.6760381

The boat is about 322 feet from the base of the lighthouse.

CHECK FOR UNDERSTANDING

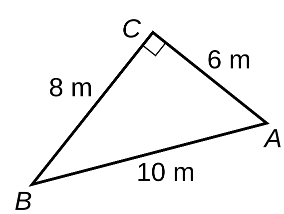
Communicating Mathematics

Read and study the lesson to answer each question.

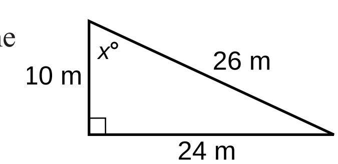
1. ***Write*** a definition for the sine and cosine ratios.
2. ***Show*** how you could use the sine or cosine to find the missing measure of one of the legs if you know the hypotenuse and an acute angle.

Guided Practice

Find each sine or cosine to the nearest tenth. Find the measure of each angle to the nearest degree.



3. cos A
4. sin A
5. $m\angle A$
6. sin B
7. cos B
8. $m\angle B$
9. Find the value of x to the nearest degree.

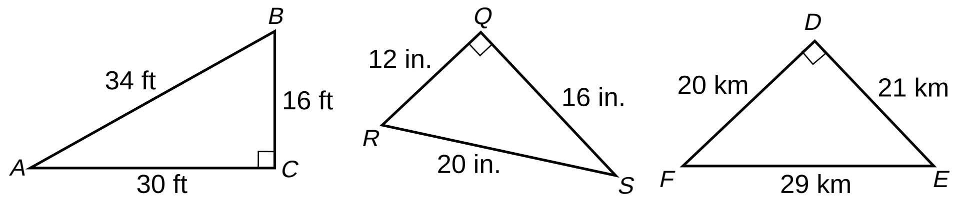


10. ***Transportation*** The end of an exit ramp from an interstate highway is 22 feet higher than the highway. If the ramp is 630 feet long, what angle does it make with the highway? Round to the nearest degree.

EXERCISES

Practice

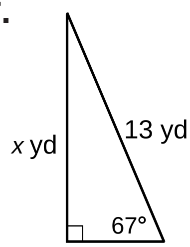
Complete each exercise using the information in the figures. Find each sine or cosine to the nearest tenth. Find the measure of each angle to the nearest degree.



11. sin A	**12.** sin B	**13.** cos A	**14.** cos B
15. $m\angle A$	**16.** $m\angle B$	**17.** sin R	**18.** cos R
19. sin S	**20.** cos S	**21.** $m\angle R$	**22.** $m\angle S$
23. sin E	**24.** cos F	**25.** $m\angle E$	**26.** $m\angle F$

Find the value of *x* to the nearest tenth or nearest degree.

27.



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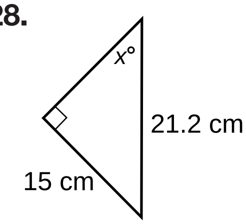
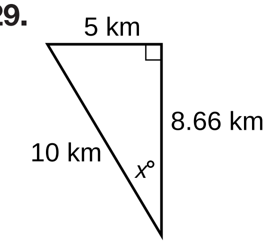


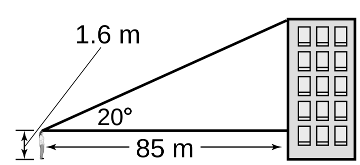
29.



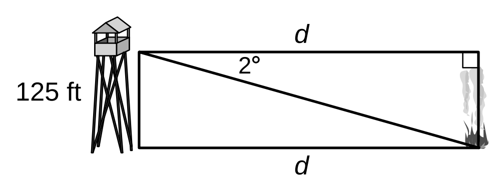
Applications and Problem Solving

30. ***Home Maintenance*** A painter props a 20-foot ladder against a house. The angle it forms with the ground is 65°. To the nearest foot, how far up the side of the house does the ladder reach?

31. ***Surveying*** A surveyor is 85 meters from the base of a building. The angle of elevation to the top of the building is 20°. If her eye level is 1.6 meters above the ground, find the height of the building to the nearest meter.



32. ***Fire Fighting*** A fire is sighted from a fire tower at an angle of depression of 2°. If the fire tower has a height of 125 feet, how far is the fire from the base of the tower? Round to the nearest foot.



33. ***Critical Thinking*** Study your answers to Exercises 11–26. Make a conjecture about the relationship between the sine and cosine of complementary angles.

Name ______________________________ Date ____

Study Guide

The Sine and Cosine Ratios

If an angle is an acute angle of a right triangle:

$$\sin A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the hypotenuse}}$$

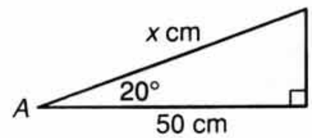
$$\cos A = \frac{\text{measure of the leg adjacent to } \angle A}{\text{measure of the hypotenuse}}$$

Example 1 Use the cosine ratio to find the value of x.

From the diagram: $m\angle A = 20°$

adjacent leg = 50 cm

hypotenuse = x cm



$\cos 20° = \frac{50}{x}$ ← adjacent leg / ← hypotenuse

$(\cos 20°)(x) = 50$

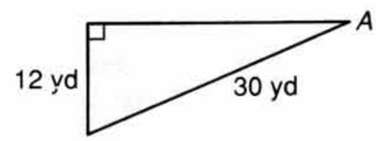
$x = \frac{50}{\cos 20°}$

Use a calculator: 50 [÷] 20 [COS] [=] 53.20888862

$x \approx$ 53 cm

Example 2 Use the sine ratio to find the measure of $\angle A$.

$\sin A = \frac{12}{30}$ ← opposite leg / ← hypotenuse



Use a calculator:

12 [÷] 30 [=] [2nd] [SIN^{-1}] 23.57817848

$m\angle A \approx 24°$

Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

1. $\sin A$ = ______

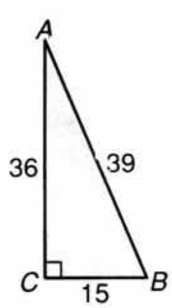
$\sin B$ = ______

$\cos A$ = ______

$\cos B$ = ______

$m\angle A$ = ______

$m\angle B$ = ______



2. $\sin L$ = ______

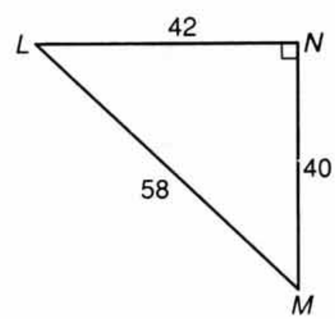
$\sin M$ = ______

$\cos L$ = ______

$\cos M$ = ______

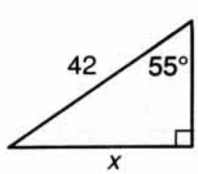
$m\angle L$ = ______

$m\angle M$ = ______

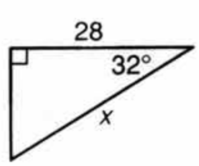


Find the value of x to the nearest tenth or to the nearest degree.

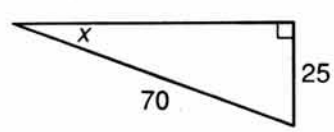
3.



4.



5.

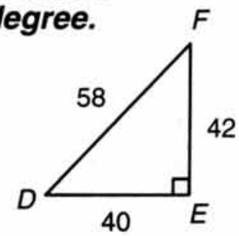


2 Practice

Name ____________________ Date ____________

The Sine and Cosine Ratios

Use the figures at the right for Exercises 1-12. Write the ratios in simplest form. Find angle measures to the nearest degree.



1. Find sin D.

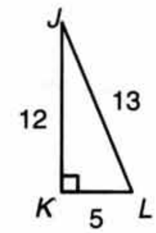
2. Find cos D.

3. Find $m\angle D$.

4. Find sin F.

5. Find cos F.

6. Find $m\angle F$.



7. Find sin L.

8. Find cos L.

9. Find $m\angle L$

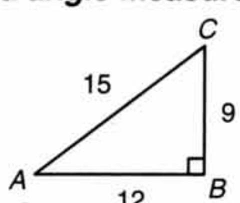
10. Find sin J.

11. Find cos J.

12. Find $m\angle J$.

Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

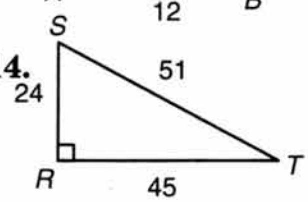
13.



$\sin A = \underline{?}$ $\cos A = \underline{?}$ $m\angle A = \underline{?}$

$\sin C = \underline{?}$ $\cos C = \underline{?}$ $m\angle C = \underline{?}$

14.

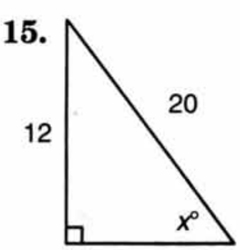


$\sin S = \underline{?}$ $\cos S = \underline{?}$ $m\angle S = \underline{?}$

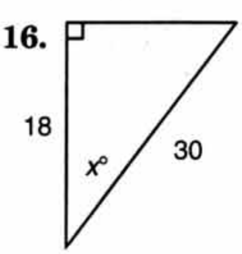
$\sin T = \underline{?}$ $\cos T = \underline{?}$ $m\angle T = \underline{?}$

Find the value of x to the nearest tenth or to the nearest degree.

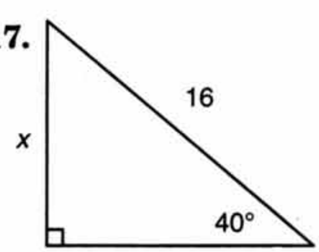
15.



16.



17.



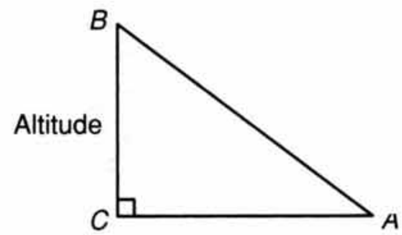
Name ______________________ Date ______________

Enrichment

Sine and Cosine Ratios

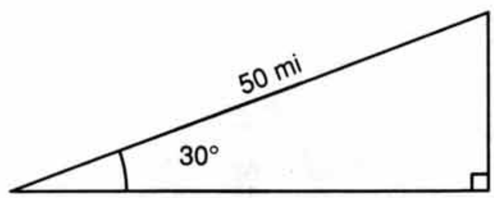
In this figure, sine A is equal to the measure of the leg opposite angle A divided by the measure of the hypotenuse.

Cosine A is equal to the measure of the leg adjacent to angle A divided by the measure of the hypotenuse.



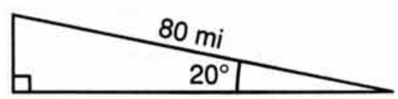
Use the sine and cosine ratios and your calculator to determine the answers to the following exercises. When necessary, round your answers to the nearest hundredth.

1. A Doppler radar station tracking a storm has located a cloud formation it suspects to be a developing tornado. The elevation angle to the cloud formation is 30 degrees and the slant range is 50 miles. Determine the horizontal distance to the suspect cloud formation and its altitude.



horizontal distance ______ mi altitude ______ mi

2. A radar installation is tracking an object of unknown origin. Data indicates that the object is hovering at a slant range of 80 miles and an elevation angle of 20 degrees. Compute the horizontal distance to the object and its altitude.



horizontal distance ______ mi altitude ______ mi

3. A plane climbing at a constant elevation angle has flown a distance of 11 miles since takeoff, as measured along its line of flight. The plane has an altitude of 3 miles. Find the elevation angle of the plane.

elevation angle ______ degrees

4. A parabolic dish antenna is aimed at a satellite with a slant range of 26,590 miles. The elevation angle is 57 degrees. Determine the altitude of the satellite.

altitude ____________ mi

Trigonometric Table

Degrees	Sine	Cosine	Tangent
0	.0000	1.0000	.0000
5	.0872	.9962	.0875
10	.1736	.9848	.1763
15	.2588	.9659	.2679
20	.3420	.9397	.3640
25	.4226	.9063	.4663
30	.5000	.8660	.5774
35	.5736	.8192	.7002
40	.6428	.7660	.8391
45	.7071	.7071	1.0000
50	.7660	.6428	1.1918
55	.8192	.5736	1.4281
60	.8660	.5000	1.7321
65	.9063	.4226	2.1445
70	.9397	.3420	2.7475
75	.9659	.2588	3.7321
80	.9848	.1736	5.6713
85	.9962	.0872	11.4301
90	1.0000	.0000	....