

Lesson 8-5

Example 1 Find a Rate of Change

POPULATION Between the years 1990 and 1995, the population of a city grew from 12,500 to 17,000. Find the rate of change in the population. Consider 1990 as year 0 and 1995 as year 5.

rate of change = slope

$$\begin{aligned} &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Definition of slope} \\ &= \frac{17,000 - 12,500}{5 - 0} && \text{Change in population over change in year} \\ &= 900 && \text{Simplify.} \end{aligned}$$

So, the rate of change in population is an increase of 900 per year.

Example 2 Compare Rates of Change

PRODUCTION The XYZ Company produced 24,000 units in its first year in operation and had increased production to 45,000 units by its fourth year in operation. The ABC Company produced 7,500 units in its first year of operation and 18,000 units by its fourth year in operation. Compare the rates of change.

$$\begin{aligned} \text{XYZ Company rate of change} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{45000 - 24000}{4 - 1} \\ &= \frac{21000}{3} && \text{Simplify.} \\ &= 7000 && \text{Simplify.} \end{aligned}$$

For each year, production increased by 7,000 units.

$$\begin{aligned} \text{ABC Company rate of change} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{18000 - 7500}{4 - 1} \\ &= \frac{10500}{3} && \text{Simplify.} \\ &= 3500 && \text{Simplify.} \end{aligned}$$

For each year, production increased by 3,500 units.

Production for the XYZ Company increased at a faster rate than production for the ABC Company.

Example 3 Write a Direct Variation Equation

Suppose y varies directly with x and $y = 8$ when $x = -2$. Write an equation relating x and y .

Step 1 Find the value of k .

$$\begin{array}{ll} y = kx & \text{Direct variation} \\ 8 = k(-2) & \text{Replace } y \text{ with } 8 \text{ and } x \text{ with } -2. \\ -4 = k & \text{Simplify.} \end{array}$$

Step 2 Use k to write an equation.

$$\begin{array}{ll} y = kx & \text{Direct variation} \\ y = -4x & \text{Replace } k \text{ with } -4. \end{array}$$

So, a direct variation equation that relates x and y is $y = -4x$.

Example 4 Use Direct Variation to Solve Problems

GARDENING The height of a tomato seedling as it grows is recorded in the table below.

Time (days)	Height (in.)	$k = \frac{y}{x}$
x	y	
4	1	0.25
6	1.5	0.25
8	2	0.25
10	3	0.30

a. Write an equation that relates time and height.

Step 1 Find the ratio of y to x for each time in days. The ratios are approximately equal to 0.25.

Step 2 Write an equation.

$$\begin{array}{ll} y = kx & \text{Direct variation} \\ y = 0.25x & \text{Replace } k \text{ with } 0.25. \end{array}$$

So, a direct variation equation that relates the time x and the height of the seedling y is $y = 0.25x$.

b. Predict how many days it will take for the seedling to be 8 inches tall.

$$\begin{array}{ll} y = 0.25x & \text{Write the direct variation equation.} \\ 8 = 0.25x & \text{Replace } y \text{ with } 8. \\ 32 = x & \text{Divide each side by } 0.25. \end{array}$$

It will take about 32 days for the seedling to grow to a height of 8 inches.