12-1

Name ______________________ Date ____________

Counting Outcomes (Pages 518–520)

1, 43, 45

QCC

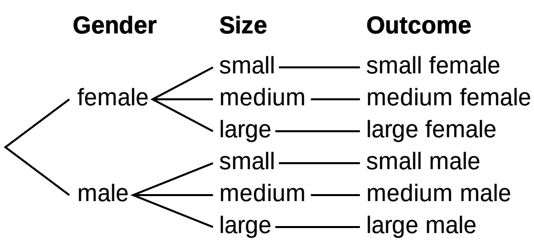
One way to find the number of possible **outcomes** is with a **tree diagram**. You can also find the total number of outcomes by multiplying with the **Counting Principle**.

Counting Principle	If event *M* can occur in *m* ways, and is followed by event *N* that can occur in *n* ways, then the event *M* followed by the event *N* can occur in $m \cdot n$ ways.

EXAMPLE

La Donna is going to adopt a puppy from the local animal shelter. The animal shelter groups their dogs by gender (male or female) and by size (small, medium, or large). Use a tree diagram and the Counting Principle to find the number of choices, or possible outcomes, that La Donna has.

Use a tree diagram.



Use the Counting Principle.

gender choices × size choices = outcomes

$2 \times 3 = 6$

There are 6 possible outcomes.

Try This Together

1. A restaurant offers three different dinner salads and six types of salad dressing. How many choices of salad with dressing are there?
HINT: Multiply.

PRACTICE

Use a tree diagram or the Counting Principle to find the number of possible outcomes.

2. Colin has a choice of a black, brown, or blue T-shirt with a choice of black, blue, or gray pants.

3. Reiko picks millet, oat, thistle, or sunflower seeds for her sparrow, finch, or dove bird feeders.

4. A restaurant offers eggs cooked three different ways with a choice of hash browns or fried potatoes.

5. **Standardized Test Practice** Olga has a choice of five different colored calligraphy pens, and plain, bond, or parchment paper. How many possible pen and paper choices does she have?

A 15 **B** 8 **C** 10 **D** 12

Answers: 1. 18 2. 9 3. 12 4. 6 5. A

Name ______________________ Date ____________

12-2 Permutations (Pages 521–523)

An arrangement or listing in which order is important is called a **permutation**.

Representing Permutations	Use $P(n, r)$ to represent a permutation. $P(n, r)$ means the number of permutations of n things taken r at a time. $P(n, r) = n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot (n - r + 1)$ For example, $P(8, 3) = 8 \cdot 7 \cdot 6$ or 336.

The notation $n!$ (n **factorial**) means the product of all counting numbers beginning with n and counting backward to 1. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1$, or 24. We define $0!$ as 1.

EXAMPLE

There are 5 runners in a 400-meter race. The first, second, and third place runners get ribbons. How many possible ways could the ribbons be awarded?

You must select 3 runners from the 5.

$P(5, 3) = 5 \cdot 4 \cdot 3$ *$n = 5$ and $r = 3$, so $n - r + 1 = 3$*

$= 60$

There are 60 ways the ribbons could be awarded.

Try These Together

Find each value.

1. $P(6, 3)$ **2.** $6!$

PRACTICE

Find each value.

3. $P(5, 5)$ **4.** $P(8, 4)$ **5.** $P(13, 5)$ **6.** $8!$ **7.** $0!$

8. $5!$ **9.** $2!$ **10.** $9!$ **11.** $P(15, 1)$ **12.** $P(10, 5)$

13. Pets How many ways can you select 5 dogs from a group of 7 to enter 5 different events at a local dog show?

14. Standardized Test Practice There are 12 preschoolers waiting to use 4 different pieces of playground equipment. How many ways can the teacher distribute the equipment to 4 students?

A 11,880   **B** 479,001   **C** 24   **D** 48

Answers: 1. 120 2. 720 3. 120 4. 1,680 5. 154,440 6. 40,320 7. 1 8. 120 9. 2 10. 362,880 11. 15 12. 30,240 13. 2,520 14. A

Name ______________________________ Date ______________

12-3 Combinations (Pages 524–527)

QCC 1, 18

An arrangement or listing in which order is not important is a **combination**.

Calculating Combinations	To find the number of combinations of n items taken r at a time, or $C(n, r)$, divide the number of permutations $P(n, r)$ by the number of ways r items can be arranged, which is $r!$. $C(n, r) = \frac{P(n, r)}{r!}$

EXAMPLES

A Find $C(3, 2)$.

$$C(3, 2) = \frac{P(3, 2)}{2!} = \frac{3 \cdot 2}{2 \cdot 1} = \frac{6}{2} \text{ or } 3$$

B Find $C(5, 3)$.

$$C(5, 3) = \frac{P(5, 3)}{3!} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = \frac{60}{6} \text{ or } 10$$

Try These Together

Find each value.

1. $C(5, 2)$ **2.** $C(12, 4)$ **3.** $C(16, 3)$ **4.** $C(8, 5)$

HINT: Find the number of permutations first, then divide by r!.

PRACTICE

Find each value.

5. $C(10, 6)$ **6.** $C(4, 2)$ **7.** $C(7, 4)$ **8.** $C(11, 5)$

9. $C(6, 3)$ **10.** $C(4, 4)$ **11.** $C(1, 1)$ **12.** $C(100, 1)$

Determine whether each situation is a permutation or a combination.

13. choosing 3 paper clips from a box of 100

14. picking 5 tennis balls from a basket of 10

15. six birds sitting on a telephone wire

16. choosing 4 colored markers from a box of 8 different colors

17. five bicycles parked at a bicycle stand for 10 bikes

18. **Purchasing** A market carries 15 flavors of gum. Nate buys three flavors of gum each time he visits the market. How many different combinations of three flavors of gum could Nate buy?

19. **Standardized Test Practice** Mr. Begay has 8 insects for students to study. How many different groups of 3 insects can a student study?

A 8 **B** 70 **C** 28 **D** 56

Answers: 1. 10 2. 495 3. 560 4. 56 5. 210 6. 6 7. 35 8. 462 9. 20 10. 1 11. 1 12. 100 13. combination 14. combination 15. permutation 16. combination 17. permutation 18. 455 19. D

Name ______________________ Date ____________

12-4 Pascal's Triangle (Pages 528–531)

QCC 1, 3, 45

Pascal's Triangle is a special mathematical pattern that is named after the French mathematician Blaise Pascal (1623–1662). You can use the pattern in Pascal's Triangle to answer questions involving combinations.

Pascal's Triangle	To find each inside number of Pascal's Triangle, add the two numbers above it.	
	1	Row 0
	1 1	Row 1
	1 2 1	Row 2
	1 3 3 1	Row 3
	1 4 6 4 1	Row 4
	1 5 10 10 5 1	Row 5
	1 6 15 20 15 6 1	Row 6

EXAMPLE

Use Pascal's Triangle to find $C(6, 3)$.

- *The first number in this combination, 6, tells you to look in row 6 of Pascal's Triangle.*
- *The second number, 3, tells you to look for the third number in row 6 of the triangle. To do this, treat the 1 at the beginning of row 6 as the zero'th number in the row.*
- *The third number in row 6 is 20. So, $C(3, 2) = 20$.*
- *You can check your answer by using the formula $C(n, r) = \frac{P(n, r)}{r!}$.*

Try These Together

Use Pascal's Triangle to find each value.

1. $C(4, 2)$
HINT: The second number in row 4 is 6.

2. $C(6, 2)$
HINT: Look at row 6.

3. $C(7, 3)$
HINT: Add a row to the triangle.

PRACTICE

Use Pascal's Triangle to find each value.

4. $C(4, 3)$ **5.** $C(8, 2)$ **6.** $C(5, 4)$

Use Pascal's Triangle to answer each question.

7. How many combinations of 6 items taken 4 at a time are possible?

8. How many ways can you choose 3 questions to answer on a 5-question essay test?

9. **School Fair** A teacher wants to pick a team of 4 students out of a group of 7 for a competition at the school fair. From how many combinations can he choose?

10. **Standardized Test Practice** How many different 5-question quizzes can Mrs. Chen make from a bank of 7 quiz items?

A 5 **B** 7 **C** 21 **D** 35

Answers: 1. 6 2. 15 3. 35 4. 4 5. 28 6. 5 7. 15 8. 10 9. 35 10. C

Name ____________________ Date ____________

12-5 Probability of Compound Events (Pages 534–537)

QCC 1, 39

When you find probability, you often have to look at two or more events, known as **compound events**. In a compound event, if the second event does not depend on the outcome of the first event, then the events are **independent**. If the outcome of one event of a compound event affects the other event, then the events are **dependent**.

Probability of Two Independent Events	The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event. $P(A \text{ and } B) = P(A) \cdot P(B)$
Probability of Two Dependent Events	If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs. $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

EXAMPLES

A What is the probability of tossing heads on a coin twice in a row?

The first coin toss does not affect the second coin toss, so these are independent events.

$P(\text{heads and heads}) = P(\text{heads}) \cdot P(\text{heads})$

$P(\text{heads and heads}) = \frac{1}{2} \cdot \frac{1}{2}$

$P(\text{heads and heads}) = \frac{1}{4}$

The probability of tossing heads twice in a row is $\frac{1}{4}$.

B A bag contains three pink and two purple marbles. What is the probability of drawing two purple marbles in a row from the bag if the first marble is not replaced?

Drawing the first marble changes the number of marbles in the bag, which changes the probability of the second event. These are dependent events.

$P(\text{purple and purple}) = P(\text{purple}) \cdot P(\text{purple after purple})$

$P(\text{purple and purple}) = \frac{2}{5} \cdot \frac{1}{4}$

$P(\text{purple and purple}) = \frac{2}{20}$ or $\frac{1}{10}$

The probability of drawing two purple marbles in a row from the bag is $\frac{1}{10}$.

PRACTICE

Twenty game cards are used. Five are red, five are blue, four are green, and six are yellow. Once a card is drawn, it is not replaced. Find the probability of each outcome.

1. two blue cards in a row

2. a green card and then a yellow card

3. **Standardized Test Practice** Sarita has four $1 bills and three $10 bills in her wallet. What is the probability that she will reach into her wallet twice, and pull out a $10 bill each time? Assume she does not replace the first bill.

A $\frac{1}{7}$ **B** $\frac{2}{7}$ **C** $\frac{6}{49}$ **D** $\frac{12}{49}$

Answers: 1. $\frac{1}{19}$ 2. $\frac{6}{95}$ 3. A

12-6

Name ______________________ Date __________

Experimental Probability

(Pages 540–543)

QCC 1, 39, 40, 43

You know that because a number cube has six possible outcomes, the probability of tossing a one is $\frac{1}{6}$. This kind of probability is called **theoretical probability**. But if you toss a cube a number of times, the fraction of times you get a one may not be exactly $\frac{1}{6}$. This is known as **experimental probability**.

EXAMPLE

Clarise conducted an experiment to find out her probability of making a free throw during a basketball game. In her experiment, she attempted 100 free throws. She hit 40 of her 100 free throws. What is her experimental probability of making a free throw?

experimental probability $= \frac{\text{number of free throws made}}{\text{number of free throws attempted}}$

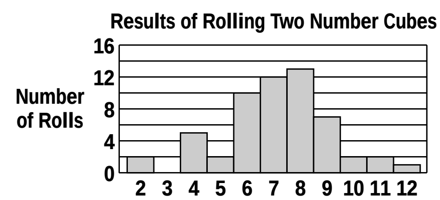
So, her experimental probability of making a free throw is $\frac{40}{100}$ *or* $\frac{2}{5}$.

PRACTICE

1. If you toss a baseball card, what is the theoretical probability that it will land with the picture face-up?

2. You have tossed the card 40 times and it lands with picture face-up 24 times. What is the experimental probability of the card landing face-up?

3. Svetlana and Lenora are playing a game with two number cubes. Based on the results from the rolls indicated on the graph, what number is Svetlana most likely to roll next?



4. Genetics Gregor grows pea plants as a hobby. Some of his pea plants always produce white flowers. Others always produce red flowers. As an experiment, Gregor pollinated a white flower with pollen from a red flower. The cross-pollinated white flower produced 8 seeds.

a. If genetic traits such as flower color are equally likely to occur, how many of those 8 seeds would you expect to grow into plants with red flowers?

b. If three of the 8 seeds grow into plants with red flowers, what is the experimental probability of a seed growing into a plant with red flowers?

5. **Standardized Test Practice** Celia has a bag of 10 marbles. Some are blue, some are yellow. She drew a marble from the bag 100 times, replacing the marble after each draw. If she drew a blue marble 78 of the 100 times, how many blue marbles are most likely in the bag?

A 3 **B** 8 **C** 7 **D** 9

Answers: 1. $\frac{1}{2}$ 2. $\frac{3}{5}$ 3. 8 4a. 4 4b. $\frac{3}{8}$ 5. B

Name ______________________ Date ____________

12-7 Using Sampling to Predict

(Pages 546–548)

QCC 1, 39, 43, 45

If you want to make a prediction about a large group of people, you may wish to use a smaller group, or **sample**, from the larger group. The large group from which you gathered your sample is known as the **population**. To make sure your information represents the population, the sample must be drawn at *random*. A random sample gives everyone the same chance of being selected.

EXAMPLES

The school math club asked several students at random what they like to eat during their afternoon snack break. Three students said they like to eat muffins, five said fruit, and one said bagels.

A What is the size of the sample?

Add the number of people who were asked.
$3 + 5 + 1 = 9$

B What percent preferred muffins?

3 out of 9 said that they like to eat muffins.
$\frac{3}{9} = \frac{1}{3}$ *or* $33\frac{1}{3}\%$

C Based on their survey, about how many of the 1,200 students in the school would prefer muffins for their afternoon snack?

$\frac{1}{3} \times 1{,}200 = 400$
So about 400 students would prefer muffins.

D Were the students the math club surveyed an appropriate sample?

The students surveyed by the math club probably were not an appropriate sample because there were so few students surveyed compared to the total number of students in the school.

PRACTICE

1. Brushy Creek Middle School is a new school with 800 students. The principal asked some students their preference for the new school mascot. The results were that 22 preferred an eagle, 36 preferred a tiger, and 42 preferred an armadillo.

a. What is the sample size?

b. What percent wanted the armadillo to be the school mascot?

2. **Biology** Every month for three years, a biologist has caught 30 fish from a lake and checked their blood for lead contamination. In the three years, she has found 270 fish with lead in their blood. If she decides to check 40 fish next month instead of 30, how many do you predict will have lead in their blood?

3. **Standardized Test Practice** A film company wants to see test-audience reactions to a new cartoon adventure film before they start advertising. Which of the following test audiences would make the best sample of the film's intended audience?

A college students
B high school students
C senior citizens
D elementary school students

Answers: 1a. 100 1b. 42% 2. 10 3. D

Name ______________________ Date ____________

12

Chapter 12 Review

Family Photo Opportunity

Eight members of Joaquin's family (including Joaquin and Irene) are eating a holiday dinner together. Joaquin has a new camera and wants to take their pictures in groups of some number (as large as possible) to make an album for those who could not come. Irene is worried that there won't be enough film for all those pictures. Help them figure out this problem.

1. How many different groups of two people can they form from those at the dinner? (Hint: A picture of Uncle Steve with Bill is the same as a picture of Bill with Uncle Steve.)

2. How many groups of three members each can be formed from the 8 people?

3. How many groups of four members each can be formed from 8 people?

4. How many groups of five members each can be formed from 8 people?

5. How many groups of six members each can be formed from 8 people?

6. Irene and Joaquin have 2 rolls of film with 36 exposures each. What is the largest size group they can use in their pictures?

Answers are located on page 125.