Key Concepts

Lesson 2

Adding Integers

Objective Teach students how to add positive and negative integers.

Note to the Teacher *In this chapter of the Student Edition, your students will learn how to add, subtract, multiply, and divide integers. Conceptually the most difficult idea for students to grasp is that of negative numbers and why there are special rules for doing arithmetic with them. So it is a good idea to begin by having a classroom discussion about negative numbers.*

History of Negative Integers

Historically, negative numbers were developed very late. Despite the fact that the ancient civilizations carried out subtraction, the recognition of negative numbers as *legitimate* numbers did not occur until the 1600s. Negative numbers are extremely useful for many kinds of problems, and in particular, they permit simplification and unification of many important processes. For instance, the concept of a negative temperature, the notion of a negative balance in a bank account, the use of negative numbers to describe a motion in a particular direction are very important examples of this idea.

Here are some examples of how negative numbers are used. Discuss them with your students and ask them if they can come up with other examples of how negative numbers might be used.

Example 1 **Express temperature as a negative number.**

Solution In the Celsius temperature scale, the freezing point of water is 0°. In the Fahrenheit scale the freezing point of water is 32°. When we want to represent a temperature in Celsius that is colder than the freezing point of water, we represent it by a negative number of degrees. For instance, a temperature that is 7° colder than the freezing point of water would be represented by the negative number $-7°$.

Example 2 **Express a change in a quantity as a negative number.**

Solution When we are measuring changes in a quantity, such as the change in temperature or a change in rainfall from one year to the next, we describe a change in which the quantity decreases as a negative number. For instance, if the temperature at noon is 20° and the temperature at 6:00 P.M. is 12°, we say that the change in temperature from noon to 6:00 P.M. is −8°. This is convenient so that we can speak of both kinds of changes (increases and decreases) on the same footing, using the same terminology.

Example 3 **Express money as a negative number.**

Solution In keeping track of the assets of corporations, a deficit (that is, a situation in which a corporation owes more money than it has in assets) can be described by saying that the company has negative assets. So, for instance, if the corporation has $3,000,000 in assets but owes $4,000,000, then we say the corporation has assets of "negative one million dollars", written as −$1,000,000.

Note to the Teacher *After this discussion about how negative numbers are used, your students should be ready to learn how to add negative integers (to positive integers). Begin by explaining the basic rules below, with examples.*

Rules for addition of negative numbers

- Adding a negative number to a positive number gives the same result as subtracting the corresponding positive number.

$$\begin{aligned} 9 + (-4) &= 9 - 4 \\ &= 5 \end{aligned}$$

- When we subtract a larger number from a smaller one, the result will be a negative number.

$$5 - 7 = -2$$

- Therefore, when we have an addition problem involving one positive number and one negative number, we may end up with a negative answer.

$$\begin{aligned} 5 + (-7) &= 5 - 7 \\ &= -2 \end{aligned} \qquad \begin{aligned} 14 + (-21) &= 14 - 21 \\ &= -7 \end{aligned}$$

- If we add two negative numbers, then the result is the opposite of what we would get if we added the corresponding positive numbers:

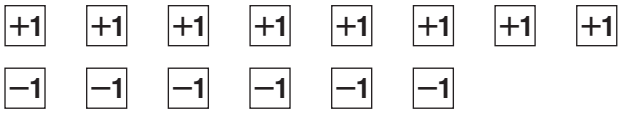
$$\begin{aligned} (-2) + (-3) &= -(2 + 3) \\ &= -5 \end{aligned}$$

Note to the Teacher *You should now explain why these rules are true. They are best explained using some manipulatives like colored blocks, or as is done in the text, colored counters.*

Why are these rules for addition true?

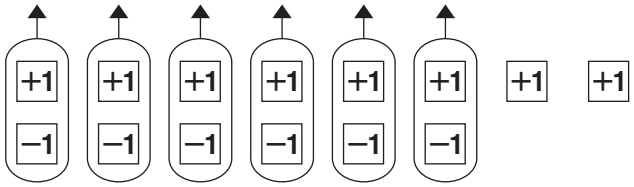
You can represent positive numbers by collections of blue blocks and negative numbers by collections of red blocks. Here the blue blocks are shown as $+1$ blocks and the red blocks as -1 blocks.

These collections of blocks represent -6 and 8, respectively. We can also combine collections of positive (blue) and negative (red) blocks. Think about this as adding a positive and a negative number.



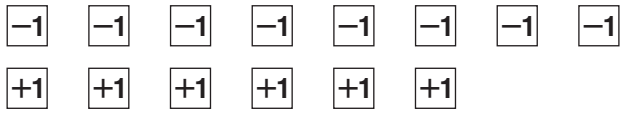
This collection represents $8 + (-6)$.

Remember that we are thinking of negative numbers as a deficit, so the collections we just drew should be thought of as a collection of 8 (dollars, say) together with a deficit of 6 dollars. Of course, the 8 dollars could be used to pay off the deficit of 6 dollars, and there would be 2 dollars left over. This can be pictured using blocks if we allow ourselves to cancel a negative (red) block and a positive (blue) block.

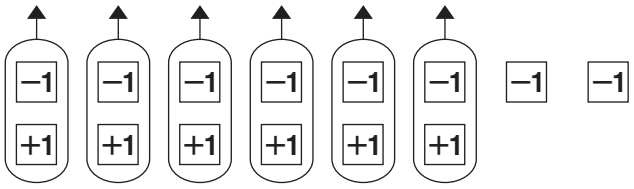


In the picture, we cancelled each negative (red) block with a positive (blue) block and removed both blocks from the picture. Whenever we have both negative (red) and positive (blue) blocks, we should cancel unlike blocks in pairs until we have a collection that consists only of blocks of a single color (kind).

Now consider $6 + (-8)$.



When we cancel pairs of positive (blue) and negative (red) blocks, we are left with two negative (red) blocks.



This means that we have a deficit of 2, so $6 + (-8) = -2$.

Example 4 **Compute $(-4) + (-6)$.**

Solution Draw -4 as a collection of four negative (red) blocks and -6 as a collection of 6 negative (red) blocks. When we merge them to add, we will get 10 negative (red) blocks.

Therefore, $(-4) + (-6) = -10$.

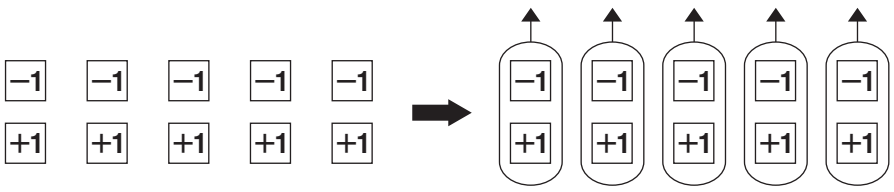
Here is a very important special case of these rules of addition.

Adding Opposites	When we add a positive number and its opposite negative number, we always get zero.

Note to the Teacher *Ask your students why they think this rule is true. Guide the discussion by considering the following examples, illustrated using manipulatives, like blocks.*

Example 5 **Demonstrate that $5 + (-5) + 0$.**

Solution Represent $5 + (-5)$ with 5 positive (blue) blocks and 5 negative (red) blocks. The opposite blocks (red and blue) cancel in pairs. We are left with no blocks at all, so the result is 0.



Have students solve elementary addition problems involving negative numbers by using colored blocks or other manipulatives.

Negative Numbers in Algebra

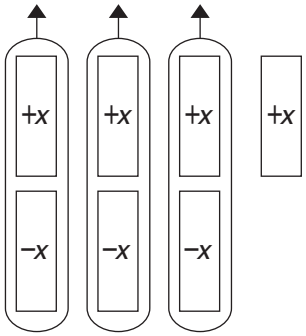
When working with variables, one can use the Distributive Property together with the rules about adding and subtracting integers to simplify expressions and equations.

Example 6 **Simplify the expression $4x + (-3)x$.**

Solution Use the distributive property.

$$\begin{aligned} 4x + (-3)x &= [4 + (-3)]x \\ &= [1]x \\ &= x \end{aligned}$$

As with actual numbers, one can think of each x and $-x$ as units that cancel out, so $-3x$ cancels out $3x$.



End of Lesson