Key Concepts

Lesson 3

Ratios and Proportions

Objective Teach students the concepts of ratio and proportion and to solve proportion problems.

Note to the Teacher *Students will be introduced to concepts of ratios and proportions. Students will also be asked to solve proportion problems that will exercise their skills with fractions, including the technique of cross multiplication. They will also use elementary algebra techniques.*

Ratios

Explain to your students the concept of *ratio*. Your students have probably seen this concept before and should not find it very difficult. Mathematically, a **ratio** is simply a fraction viewed as a division of two numbers. Its importance is that it is used to *compare* two numbers (the numerator and the denominator of the fraction). For example, the ratio of x to y is simply the fraction $\frac{x}{y}$. A ratio can also be written as

$$x \text{ to } y \qquad \text{or} \qquad x{:}y.$$

Use the following example to show how a ratio compares two numbers.

Example 1 **In Ms. Cunningham's class there are 18 girls and 14 boys. Write the ratio of boys to girls.**

Solution The ratio of boys to girls in the class can be written as

14 to 18 or 14:18 or $\frac{14}{18}$

When simplified, this ratio can also be expressed as

7 to 9 or 7:9 or $\frac{7}{9}$

Proportions

An equation that states that two ratios are equal is called a **proportion.** The equation $\frac{14}{18} = \frac{7}{9}$ is a proportion.
This proportion can also be written as 14:18 = 7:9

More generally a proportion will often involve variables. Solving these problems usually involves elementary algebra, because they involve solving for the value of a variable. To solve these kinds of problems we use a process called cross multiplying. The cross multiplying fact should be explained carefully to your students.

Cross Multiplication Fact	Suppose y and b are not zero. Then $\frac{x}{y} = \frac{a}{b}$ occurs exactly when $xb = ya$.

Example 1 **Solve $\frac{3}{5} = \frac{a}{15}$ for a.**

Solution Use the cross multiplication fact and solve the resulting equation.

$$\frac{3}{5} = \frac{a}{15}$$

$$3(15) = 5(a)$$

$$45 = 5a$$

$$9 = a$$

Note to the Teacher *Lead a discussion in class about why the cross multiplication fact holds true. Ask your students why they think it works. Lead them to understand that it is really derived from the subtraction of fractions.*

Why is the cross multiplication fact true?

Notice that $\frac{x}{y} = \frac{a}{b}$ occurs exactly when $\frac{x}{y} - \frac{a}{b} = 0$. Subtract these fractions by finding a common denominator.

$\frac{x}{y} - \frac{a}{b} = 0$ occurs exactly when $\frac{xb - ya}{yb} = 0$.

Remember that a fraction equals 0 only when its numerator equals 0.

So for this equation, $\frac{xb - ya}{yb} = 0$ when $xb - ya = 0$ or when $xb = ya$.

This is the result of cross multiplying $\frac{x}{y} = \frac{a}{b}$.

Note to the Teacher *The above discussion about why the cross multiplication fact is true is a very important example of mathematical reasoning. Namely, it uses a computational technique (subtraction of fractions) to derive a general principle (cross multiplication fact). In order to solidify this concept you should do many examples of how this technique is applied. Do the following examples in class and assign several others to your students to work on individually or in small groups.*

Example 2 **In the New Hampshire primary election, the ratio of Republicans to Democrats voting was 5:4. It was reported that 200,000 Democrats voted. How many Republicans voted?**

Solution Let R be the number of Republicans that voted. Write a proportion from the information in the problem.

$$\frac{5}{4} = \frac{R}{200{,}000}$$

Use the cross multiplication fact and solve the resulting equation.

$$\frac{5}{4} = \frac{R}{200{,}000}$$

$$5(200{,}000) = 4(R)$$

$$1{,}000{,}000 = 4R$$

$$250{,}000 = R$$

Thus, 250,000 Republicans voted.

Example 3 **When cooking rice, the ratio of water to uncooked rice should be 2:1. To make 6 cups of cooked rice, how many cups of water and uncooked rice should be used?**

Solution Let W represent the cups of water needed and R the cups of uncooked rice needed. We know that the ratio of W to R must be 2:1. So we can write a proportion.

$$\frac{W}{R} = \frac{2}{1}$$

Cross Multiply $W \cdot 1 = R \cdot 2$

$$W = 2R$$

We know that the total amount of cooked rice we want is 6 cups. That means that the total volume of the water and the uncooked rice should equal 6 cups.

$$W + R = 6$$

Substitute $2R$ from the equation above into this equation.

$$W + R = 6$$

$$2R + R = 6$$

$$3R = 6$$

$$R = 2$$

This means we need 2 cups of rice.
Since $W + R = 6$, $W = 4$. Thus, we also need 4 cups of water.

Notes to the Teacher *Ratio and proportion problems are fun for students, because they deal with real-world situations. However, they tend to be difficult because of the basic algebra that is used in solving them. So, it will help if your students work together in small groups on word problems involving ratios and proportions. Have each group present their solutions to the rest of the class. Preparing class presentations helps to solidify the students' understanding of the methods use.*