Key Concepts

Lesson 4

Percent of Change

Objective Teach students how to compute the percent of change of a certain quantity.

Note to the Teacher *When a certain quantity changes, it is often important to know by what percent did it change? For example, the statement there was an inflation of 5% in housing prices in the last year, means that the cost of housing went up 5% during the year. In this lesson students will learn how to compute the percent of change.*

Start with the following example.

Computing Percent of Change

There are two methods you should explain to your students.

Example 1 **Find the percent of change from 125 to 175.**

Solution Method 1 Find the difference of the quantities (the *change*) and compute what percent of the original quantity the difference is.

In this example, the difference of the quantities is 175 − 125 = 50. So the quantity increased by 50. We then need to find out what percent the difference 50 is of the original quantity 125.

Let x represent this percent.

50 is x% of 125

Substitute a multiplication sign for the word *of,* we have the equation $50 = x\% \cdot 125$. Solve this equation for x.

$$50 = x\% \cdot 125$$
$$\frac{50}{125} = \frac{x}{100}$$
$$\frac{50 \cdot 100}{125} = x$$
$$40 = x$$

So the percent change from 125 to 175 is 40%. Since the quantity *increased* from 125 to 175, we say that the percent increase is 40%.

Method 2 When you are asking for the percent change from 125 to 175, you are essentially asking what percent of 125 is 175. Let y represent this percent.

$$\frac{175}{125} = \frac{y}{100}$$
$$\frac{175 \cdot 100}{125} = y$$
$$140 = y$$

This says that 175 is 140% of 125. Since 100% of a number is equal to that number, the percent increase is 40%.

Notice that a shortcut to this calculation is simply to divide the new amount by the original amount.

$$\frac{175}{125} = 1.4$$

Then subtract 1 and express the resulting decimal as a percent.

$1.4 - 1 = 0.4$ $\qquad$ $0.4 = 40\%$

Here is another example.

Example 2 **The usual price of a hotdog at Susan's Diner is $2.50. Last Sunday, they had a 20% off sale on hotdogs. How much did Susan charge for the hotdog on Sunday?**

Solution This problem can be solved in two steps. Step 1 is to find the amount of the discount. Step 2 is to find the price of the hotdog.

Step 1 Let D be the discount. We are told that the discount is 20% of the usual price, which is $2.50. Write an equation to find the discount.

$D = 20\%$ of $\$2.50$
$D = 0.20 \cdot \$2.50$
$D = \$0.50$

So the discount was 50 cents.

Step 2 To find the price of the hotdog on Sunday, subtract the discount D from the usual price of $2.50.

Price of Sunday hotdog $= \$2.50 - \$0.50$
$= \$2.00$

Point out to your students that a sale represents a percent *decrease* since the amount of the price decreased. This problem computes that a 20% decrease in $2.50 is $2.00.

Here is a similar example involving sales tax. This represents a percent *increase.*

Example 3 **When Cori was in France with her family on vacation, she wanted to buy some CDs. The price of the CD was 95 francs each, but when she tried to pay for them, the cashier told her that there was a 14% value-added tax (VAT). Including the tax, how much did Cori actually have to pay for each CD?**

Solution Like Example 2, we can solve this problem in two steps. Step 1 is to find out the amount of tax. Step 2 is to find out the total cost of each CD.

Step 1 Let T represent the amount of tax on each CD. We are told that this amount is 15% of the price of the CD, which is 95 francs. Write an equation to find the amount of tax.

$$\begin{aligned} T &= 15\% \text{ of } 95 \\ &= 0.15 \cdot 95 \\ &= 14.25 \end{aligned}$$

The tax was 14.25 francs.

Step 2 To find the actual price Cori had to pay for each CD, we add the tax amount to the price of the CD.

$$\begin{aligned} \text{total price} &= 95 \text{ francs} + 14.25 \text{ francs} \\ &= 109.25 \text{ francs} \end{aligned}$$

Now present the following example. This problem is a bit more challenging because we do not know the quantity from which to take a percent. Learning to solve problems like this will strengthen your students' algebra skills.

Example 4 **Mrs. Chou weighed 150 pounds at the doctor's office. The doctor told her that she was 20% over the recommended weight for her height and age. What is Mrs. Chou's recommended weight?**

Solution Since Mrs. Chou's recommended weight is what we are trying to find, let us use W to represent it. We know Mrs. Chou's current weight, 150 pounds, is 20% more than W. In other words, 150 is a 20% increase of W. Write this in an equation.

$$150 = W + (20\% \text{ of } W)$$
$$150 = 1W + 0.2W$$
$$150 = (1 + 0.2)\ W \qquad \textit{Use the Distributive Property.}$$
$$150 = 1.2W$$
$$\frac{150}{1.2} = W$$
$$125 = W$$

Mrs. Chou's recommended weight for her height and age is 125 pounds.

After working this problem, give your students the following similar, but definitely different problem.

Example 5 **Suppose Mrs. Chou's doctor told her to lose 20% of her present weight, which is 150 pounds. How much would she weigh then?**

Solution Let A be the amount of weight Mrs. Chou needs to lose. This amount is 20% of 150 pounds. Write an equation.

$A = 20\%$ of 150
$A = 0.2 \cdot 150$
$= 30$

Mrs. Chou needs to lose 30 pounds. After losing 30 pounds, her weight would be $150 - 30$ or 120 pounds.

Note to the Teacher *Now have a class discussion about these Examples 4 and 5. In Example 4, Mrs. Chou's recommended weight was 125 pounds. In Example 5, Mrs. Chou's weight after dieting would be 120 pounds. Why were there two different answers if both problems talked about 20% of her weight?*

Make sure students understand that the 20% in each problem represented 20% of two different quantities. In Example 4, the doctor said that Mrs. Chou was 20% over her recommended weight, so the extra weight was 20% of the calculated 125 pounds. That is, $20\% \cdot 125 = 25$ pounds. In Example 5, the doctor said she should lose 20% of her weight. In this problem the extra weight is 20% of 150 pounds, or 30 pounds. Even though the problems sound similar they are not the same. They involve taking 20% of two different quantities.