Key Concepts

Lesson 13

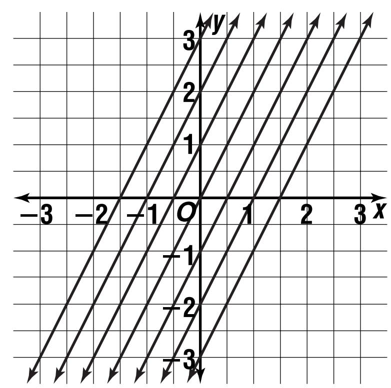
Integration: Geometry Parallel and Perpendicular Lines

Objective Guide students in understanding the fact that parallel lines have the same slope and that if two lines are perpendicular, then their slopes are negative reciprocals of each other.

Note to the Teacher *This lesson builds on Lesson 6-5A, where students explored families of graphs, including families in which the slopes were all the same, so the lines were parallel. It is an excellent idea to remind the class of that exploration before beginning this lesson.*

Slope and Parallel Lines

Begin the lesson by drawing the lines whose equations are $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, $y = 2x + 3$, $y = 2x - 1$, $y = 2x - 2$, and $y = 2x - 3$.



Ask students what the slopes of these lines are. Students should realize that they can find the slopes by using one of two methods.

Method 1: Choose points on the lines.

Method 2: Recognize that the equations are in slope-intercept form, so the coefficients of x are the slopes.

Note to the Teacher *Reinforce slope-intercept form by pointing out this second method of finding the slope.*

The conclusion is that all of the slopes have the value 2.

Next, ask students what they can say about the lines. If the answer "They are parallel" is given, ask the question, "What does parallel mean?"

Key Idea	Two lines in a plane are parallel if they never meet.

Point out that if we take any two different lines in the family given above, they never meet because they "stay the same distance apart" as we move out along the lines. This example should allow you to state the following key idea.

Key Idea	Two lines are parallel if and only if they have the same slope.

Now, work several examples, or better, have the class work them. Here are some examples. There are other excellent examples in Lesson 6-6 of the Student Edition.

Example 1 **Are the lines given by the equations $2x + 3y = 5$ and $4x + 6y = 9$ parallel? Why or why not?**

Solution First, write each equation in slope-intercept form.

$2x + 3y = 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3}$ $\qquad$ $4x + 6y = 9 \Rightarrow y = -\frac{2}{3}x + \frac{3}{2}$

They have the same slope, $-\frac{2}{3}$, so the lines are parallel.

Example 2 **Find a line through the point at (7, 5) that is parallel to the line given by the equation $y = 3x - 2$.**

Solution The given line has slope 3, since it is in slope-intercept form, so we need a line of slope 3 containing the point at (7, 5). This is given by the equation in point-slope form $y - 5 = 3(x - 7)$.

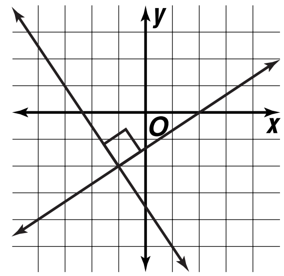
Example 3 **Line *m* contains the points at (1, 2) and (4, 7). Line *n* contains the points at (0, 0) and (5, 8). Are these lines parallel?**

Solution slope of line $m = \frac{7-2}{4-1}$ or $\frac{5}{3}$ $\qquad$ slope of line $n = \frac{8-0}{5-0}$ or $\frac{8}{5}$

Since these slopes are not equal, the lines are not parallel.

Slope and Perpendicular Lines

Remind students that perpendicular lines make a right angle with each other. Draw a pair of perpendicular lines on the chalkboard.



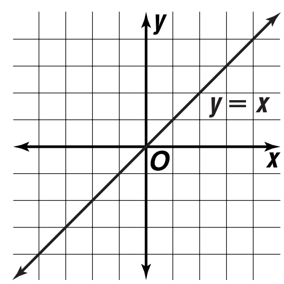
Next, ask the question, "If we know the slope of one of the lines, what will be the slope of the other line?"

Note to the Teacher *There is a very simple explanation for the answer, which we will show here. If there isn't time, simply state the answer and verify it for some examples.*

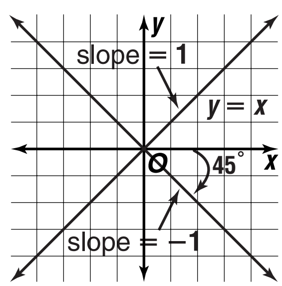
Key Idea	If the slope of one line is m, then the slope of the other line is $-\frac{1}{m}$, the negative reciprocal of the slope of the first line.

Example 4 **A line is perpendicular to the line $y = x$. What is its slope?**

Solution First, use the key idea. The slope of the line $y = x$ is 1, so the slope of any perpendicular line is $-\frac{1}{1}$ or -1. Draw the line $y = x$.

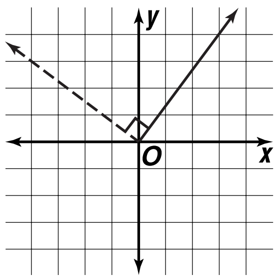


Next, draw a perpendicular line through the origin, and observe that it slopes downward at an angle of 45 degrees, and indeed has a slope of -1.

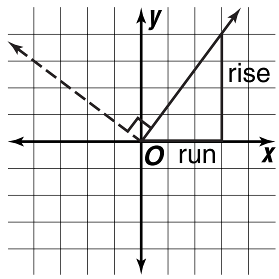


Here is a geometric explanation of the previous key idea.

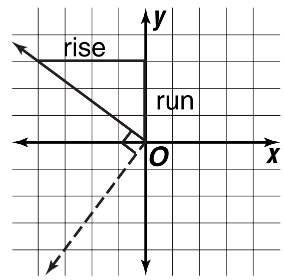
Suppose there are two perpendicular lines.



To get the slope of the first line, draw a triangle whose legs are the rise and the run.



Rotate the whole figure so that the solid line is where the dashed line used to be.



It is important to note that rise and run here refer to the original rise and run of the solid line. From the graph, this means that the rise of the dashed line is the negative of the run of the solid line. (It is negative since the dashed line is sloping downwards). Also, the run of the dashed line is the same as the rise of the solid line.

$$\text{slope of the dashed line} = \frac{-\text{run}}{\text{rise}}$$

$$= -\frac{1}{\frac{\text{rise}}{\text{run}}} \text{ or } -\frac{1}{m}$$