Key Concepts

Lesson 15

Solving Multi-Step Inequalities

Objective Teach students to solve linear inequalities involving more than one operation.

Note to the Teacher *This lesson is an extension of the methods students have learned to solve linear equations. The methods used are the Addition and Multiplication Properties of Inequalities, which parallel the corresponding properties for equalities. There are two significant distinctions that must be remembered. The first is that when multiplying or dividing an inequality by a negative number, the direction of the inequality symbol must be changed. The other is that the term "solve" is used a bit differently when speaking of inequalities. In equations, "solve" means to find a particular number that is the only solution to the equation. However, the result of solving an inequality is never a single number but rather the description of a set, such as $x \geq 5$ or $22 \leq x \leq 7$. It is very important to explain this distinction to the class.*

Solving Inequalities

Begin by reviewing the properties of inequalities.

Addition and Subtraction Properties of Inequalities	Adding or subtracting a fixed number to each side of an inequality produces an equivalent inequality. Any solution of either inequality is a solution of the other.

$$\begin{aligned} x - 2 &\leq 1 \\ x - 2 + 2 &\leq 1 + 2 \quad \textit{Add 2 to each side.} \\ x &\leq 3 \end{aligned}$$

Adding 2 to each side of the original inequality shows that $x - 2 \leq 1$ is equivalent to $x \leq 3$.

Multiplication and Division Properties of Inequalities	• Multiplying or dividing each side of an inequality by the same positive number produces an equivalent inequality. • Multiplying or dividing by the same negative number produces an equivalent inequality if the direction of the inequality symbol is reversed.

$3x \geq 12$

$x \geq 4$ *Divide each side by 3.*

So, $3x \geq 12$ is equivalent to $x \geq 4$.

$-x \geq 5$

$x \leq -5$ *Multiply each side by -1.*

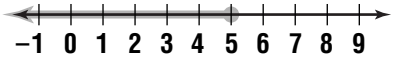
So, $-x \geq 5$ is equivalent to $x \leq -5$.

Graphing Solutions of Inequalities

Emphasize the fact that solving an inequality means describing a set, not just finding a number. This set is the **solution set** of the problem. To visualize the solution set, draw it on the number line. To do this, draw the number line and shade the part of the number line corresponding to the solution set. The solution set to a single inequality will be a half line that may or may not include the endpoint. When the endpoint is included, draw a solid dot for the endpoint. When the endpoint is not included, draw a circle.

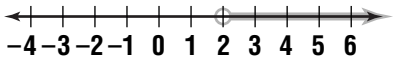
Example 1 **Represent the solution set of $x \leq 5$ on a number line.**

Solution The solution set of $x \leq 5$ is represented by the set of points to the left of 5 on the number line. It does include the endpoint, since $x = 5$ is included.



Example 2 **Represent the solution set of $x > 2$ on a number line.**

Solution The solution set of $x > 2$ is a half line represented by the set of points to the right of 2 on the number line. It does not include the endpoint, since $x = 2$ is not included.



In previous lessons, students solved one-step problems using these properties. In this lesson, multi-step problems (problems in which more than one property is used or in which one property is used more than once) are introduced. The best way to introduce these problems is by doing one or two on the chalkboard, and then asking the class to work on some additional exercises.

Note to the Teacher *Students should have practice working each of the following types of exercises.*

(1) exercises in word problem format
Example: Joe says, "In two years I will be more than half as old as Jean." Jean is 18 years old now. What are the possibilities for Joe's age?

(2) exercises in which students graph the solution sets on the number line

*(3) exercises that include a **replacement set**, a set from which the solutions must be chosen. This set could be the set of whole numbers or the set of integers.*