# Key Concepts

Lesson 18

## Graphing Systems of Equations

**Objective** Guide students in understanding the geometric interpretation of systems of two linear equations in two variables, and in solving them graphically.

**Note to the Teacher** *This chapter provides a great opportunity for students to discover ideas about systems of equations. Make sure that students graph several systems for themselves, at least one of which has no solution, and at least one of which has infinitely many solutions.*

### Systems of Linear Equations

**Note to the Teacher** *The following example is of a type that occurs frequently. It reinforces students' skills in solving word problems. Make sure to emphasize how to set up the problem.*

**Example** **Apples sell for \$2 per pound and oranges sell for \$3 per pound. Suppose Ben bought 5 pounds of fruit at a cost of \$12. How many pounds of each kind of fruit did Ben buy?**

**Solution** Write equations describing the information that is known. The first step is to assign variables to each of the unknown quantities, in this case the number of pounds of apples and the number of pounds of oranges Ben bought.

Let $A$ = the number of pounds of apples.

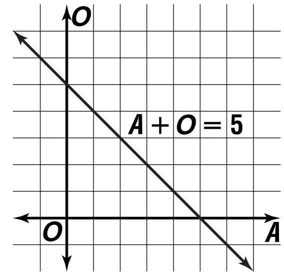
Let $O$ = the number of pounds of oranges.

Ben bought a total of 5 pounds of fruit. Write the following equation to represent this information.

$$\underbrace{\textit{Pounds of apples}}_{A} \quad \textit{plus} \quad \underbrace{\textit{pounds of oranges}}_{O} \quad \textit{equals} \quad \underbrace{\textit{total pounds of fruit.}}_{5}$$

$$A + O = 5$$

So $A + O = 5$ represents the amount of fruit that Ben bought. Note that this is a linear equation in standard form, in the variables $A$ and $O$. Its graph is shown at the right.

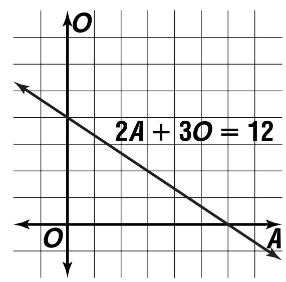



Ben spent \$2 for each pound of apples and \$3 for each pound of oranges. So the cost of apples is $2 \times$ (number of pounds of apples) or $2A$ and the cost of oranges is $3 \times$ (number of pounds of oranges) or $3O$. The total number of dollars Ben spent is \$12.

$$\underbrace{\textit{Cost of apples}}_{2A} \quad \textit{plus} \quad \underbrace{\textit{cost of oranges}}_{3O} \quad \textit{equals} \quad \underbrace{\textit{total cost of fruit.}}_{12}$$

$$2A + 3O = 12$$

This means that $2A + 3O = 12$ holds true for the variables $A$ and $O$. This is also a linear equation in standard form in the variables $A$ and $O$. The graph of this equation is shown at the right.

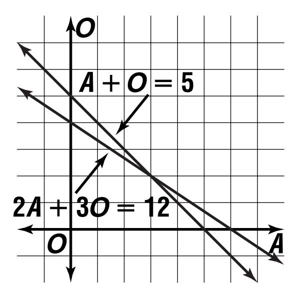



| **Key Idea** | A pair of linear equations like $A + O = 5$ and $2A + 3O = 12$ is called a **system of linear equations**. Its solution set is the set of all values of the two variables that satisfy both equations. |
|---|---|

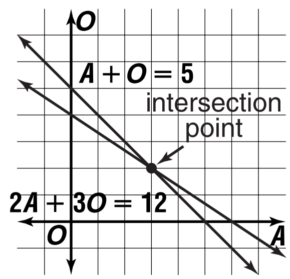
So $A$ and $O$ must satisfy both $A + O = 5$ and $2A + 3O = 12$, or satisfy the equations simultaneously.

A good way to understand the solution to a system of equations is to graph both equations on the same axes, as shown at the right.

Ask students, "What information can be found from this graph?"



First, the set of points on the graph of $A + O = 5$ is the set of all points that satisfy that equation, or the set of all ordered pairs $(A, O)$ for which the equation holds true. In the same way, the graph of $2A + 3O = 12$ is the set of all points for which that equation holds true. Therefore, the solution to the system of equations is the set of all ordered pairs $(A, O)$ for which both equations hold true. The solution to the system must correspond to a point that lies on both graphs, or that lies on the intersection of the graphs.




There is only one intersection point, a point that lies on both graphs. It is the point with coordinates (3, 2), or $A = 3$ and $O = 2$. This means that Ben bought 3 pounds of apples and 2 pounds of oranges.

**Note to the Teacher** *Have students graph these lines and find the coordinates of the intersection point themselves. It is best to have them try some additional examples that are not word problems, so they get the idea. For example, have students solve graphically the system $x + 2y = 1$ and $2x + y = 5$. Point out that in this case also, there is only one solution.*

Ask students, "Is there always only one solution to a system of linear equations?" Suggest that students think about the problem graphically. A system of linear equations corresponds to a pair of straight lines. When there are two straight lines, there are three possibilities.

| **(1)** The lines intersect in exactly one point. | **(2)** The lines are parallel, and do not intersect at all. | **(3)** The lines are actually the same line. |
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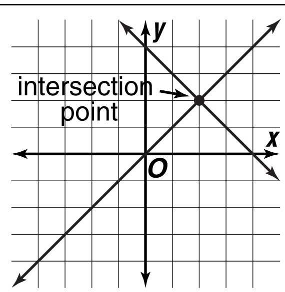
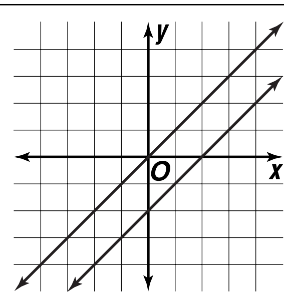
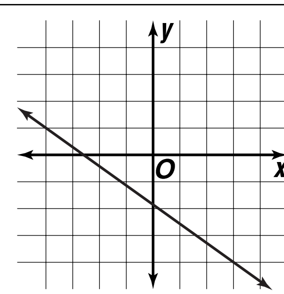
|  |  |  |
| In this case, there is **exactly one** solution, corresponding to the point of intersection. | In this case, there are **no** solutions, since the graphs do not meet. | In this case, there are **infinitely many** solutions, since every point on one of the lines lies on both lines. |

**Note to the Teacher** *It is a good idea to have all these graphs on the chalkboard simultaneously and to include the verbal description of the solution sets (one point, empty, or infinite). The following exercises provide practice in solving each of the three types of systems. Either present them as examples, or have students work them independently.*

## Exercises

**Determine whether each system has *one solution, no solution,* or *infinitely many solutions* by graphing. If the system has one solution, name it.**

**1.** $y = 2x + 4$
$y = 2x - 1$
**no solution**

**2.** $y = -x - 3$
$y = x + 3$
**(−3, 0)**

**3.** $x + 4y = -8$
$y = -\frac{1}{4}x - 2$
**infinitely many solutions**

End of Lesson