Key Concepts

Lesson 23

Dividing Monomials

Objective Introduce the notion of dividing monomials, and the idea of zero and negative numbers as exponents.

Note to the Teacher *It is a good idea to allow students to experiment with computing quotients of powers of 2 in order to discover the formula for division of powers. Also, when introducing negative powers, make it clear that the definition is based on patterns, but that it does not mean that we can really take a "product of a negative number of factors."*

Dividing Powers of the Same Number

To understand division of powers, look at the table at the right. It shows the results when 64, which equals 2^6, is divided by various powers of 2.

Ask students if they see a pattern in these numbers. Lead the class to the conclusion that the result is found by subtracting the exponents. Next, do an example with a variable.

n	$\frac{2^6}{2^n}$	$\frac{64}{2^n}$
1	$\frac{2^6}{2^1}$	$\frac{64}{2} = 32$ or 2^5
2	$\frac{2^6}{2^2}$	$\frac{64}{4} = 16$ or 2^4
3	$\frac{2^6}{2^3}$	$\frac{64}{8} = 8$ or 2^3
4	$\frac{2^6}{2^4}$	$\frac{64}{16} = 4$ or 2^2
5	$\frac{2^6}{2^5}$	$\frac{64}{32} = 2$ or 2^1
6	$\frac{2^6}{2^6}$	$\frac{64}{64} = 1$ or 2^0

Example 1 **Divide x^4 by x^2.**

Solution Expand x^4 into $x \cdot x \cdot x \cdot x$ and x^2 into $x \cdot x$. Then use these expressions to write a fraction.

$$\frac{x \cdot x \cdot x \cdot x}{x \cdot x}$$

Now cancel two of the x's from both numerator and denominator.

$$\frac{x \cdot x \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = \frac{x \cdot x}{1} = x^2$$

Note to the Teacher *Make sure to explain that "cancellation" just means recognizing that*

$$\frac{x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x}{1} \cdot \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = x \cdot x \cdot 1.$$

Students should understand that canceling is just shorthand for a process involving the properties of fractions. Also, point out that any number raised to the power 1 is that number itself.

Quotient of Powers	If we divide a power of a number or variable by another (smaller) power of the same number or variable, the result is the original number raised to the power given by the difference of the two original powers. $\frac{x^m}{x^n} = x^{m-n}$ for $m > n$

Write the quotient and expand the numerator and the denominator into products of x's.

$$\frac{x^m}{x^n} = \frac{\overbrace{x \cdot x \cdot \ldots \cdot x}^{m \text{ factors}}}{\underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}} = \frac{\overbrace{x \cdot x \cdot \ldots \cdot x}^{m-n \text{ factors}}}{1} \cdot \frac{\overbrace{x \cdot x \cdot \ldots \cdot x}^{n \text{ factors}}}{\underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}} = \overbrace{x \cdot x \cdot \ldots \cdot x}^{m-n \text{ factors}} \cdot 1 = x^{m-n}$$

This shows that the formula for the quotient of powers is valid.

Example 2 **Simplify $\frac{x^5}{x^2}$.**

Solution $\frac{x^5}{x^2} = x^{5-2}$

$= x^3$

Example 3 **Simplify $\frac{243}{81}$.**

Solution $\frac{243}{81} = \frac{3^5}{3^4}$

$= 3^{5-4}$

$= 3^1$ or 3

Example 4 **Simplify $\frac{y^{100}}{y^{25}}$.**

Solution $\frac{y^{100}}{y^{25}} = y^{100-25}$

$= y^{75}$

Example 5 **Simplify $\frac{z^{10}}{z}$.**

Solution $\frac{z^{10}}{z} = \frac{z^{10}}{z^1}$

$= z^{10-1}$

$= z^9$

This formula can be used to simplify any quotient of monomials.

Example 6 Simplify $\dfrac{a^5b^6c}{ab^3}$.

Solution $\dfrac{a^5b^6c}{ab^3} = \dfrac{a^5}{a} \cdot \dfrac{b^6}{b^3} \cdot \dfrac{c}{1}$

$= a^{5-1}b^{6-3}c$

$= a^4b^3c$

Example 7 Simplify $\dfrac{4x^7y^6}{7x^2y^2}$.

Solution $\dfrac{4x^7y^6}{7x^2y^2} = \dfrac{4}{7} \cdot \dfrac{x^7}{x^2} \cdot \dfrac{y^6}{y^2}$

$= \dfrac{4}{7}x^{7-2}y^{6-2}$

$= \dfrac{4}{7}x^5y^4$

Negative Numbers and Zero in the Exponent

Point out that we have only defined exponentiation by positive integers, not for zero or for negative numbers. Remind students that when $m > n$,

$$\frac{a^m}{a^n} = a^{m-n}.$$

If we let $m = n$, then we get

$$\frac{a^m}{a^m} = a^{m-m} = a^0.$$

On the other hand, $\dfrac{a^m}{a^m} = 1$, since any number divided by itself is 1. This leads to the following definition.

Zero Exponent	a^0 is defined to be equal to 1.

What would the formula $\dfrac{a^m}{a^n} = a^{m-n}$ suggest if $m < n$? For example, let $m = 1$ and $n = 2$.

$$\frac{a^m}{a^n} = \frac{a^1}{a^2}$$

$$= a^{1-2}$$

$$= a^{-1}$$

On the other hand,

$$\frac{a^1}{a^2} = \frac{a}{a \cdot a} = \frac{a}{a} \cdot \frac{1}{a} = 1 \cdot \frac{1}{a} = \frac{1}{a}.$$

So, the formula suggests that a^{-1} should be defined to be $\dfrac{1}{a}$. More generally, it suggests that a^{-n} should be defined to be $\dfrac{1}{a^n}$.

Negative Exponents	a^{-n} is defined to be equal to $\frac{1}{a^n}$.

Note to the Teacher *At this point, students may be confused by the way the formula was used to generate a definition for negative powers. Point out that it was used only as a guide to suggest what a definition should be, but that once we make the definition, we can work with these exponents in exactly the same way that we did with positive exponents. Emphasize that with this definition, these exponents satisfy all the laws of exponents that students studied in Lesson 9-1.*

The definition of negative exponents can be used to simplify quotients of monomials.

Example 8 **Simplify $\frac{3a^2b^9c^7}{2a^5b^3c^7}$.**

Solution $\frac{3a^2b^9c^7}{2a^5b^3c^7} = \frac{3}{2} \cdot \frac{a^2}{a^5} \cdot \frac{b^9}{b^3} \cdot \frac{c^7}{c^7}$

$= \frac{3}{2}a^{2-5}b^{9-3}c^{7-7}$

$= \frac{3}{2}a^{-3}b^6c^0$

$= \frac{3b^6}{2a^3}$

End of Lesson