Key Concepts

Lesson 24

Adding and Subtracting Polynomials

Objective Introduce addition and subtraction of polynomials.

Note to the Teacher *This lesson is not difficult conceptually, but it is important that students get sufficient practice in addition and subtraction of polynomials.*

Adding Polynomials

Introduce the notion of addition of polynomials by using a word problem.

Example 1 **Anne and Joe both make square tablecloths. Anne charges $1.00 for every square foot of area plus $2.00 for every foot of perimeter. Joe charges $1.50 for every square foot of area. How much would it cost to buy identical tablecloths, with sides of length x, from both Anne and Joe?**

Solution The answer will be given in terms of x, since we do not know the side length. The area of Anne's tablecloth is the square of the length of a side, or x^2. The perimeter is 4 times the length of one side, since the square has four identical sides. So, the perimeter is $4x$. Since Anne charges $1.00 for every square foot of area and $2.00 for every foot of perimeter, we can write an expression to represent the cost.

$$\begin{aligned}\text{cost of Anne's tablecloth} &= \$1.00 \cdot \text{area} + \$2.00 \cdot \text{perimeter} \\ &= \$1.00 \cdot x^2 + \$2.00 \cdot 4x \\ &= x^2 + 8x \text{ dollars}\end{aligned}$$

Notice that the cost is a **polynomial**. A polynomial is a monomial or a sum of monomials. Since Joe only charges a dollar for every square foot of area, the cost of his tablecloth is given below.

$$\begin{aligned}\text{cost of Joe's tablecloth} &= \$1.50 \cdot \text{area} \\ &= \$1.50 \cdot x^2 \\ &= 1.5x^2 \text{ dollars}\end{aligned}$$

Again, this is a polynomial. Now add these two polynomials together to find the total cost of both tablecloths.

$$(x^2 + 8x) + (1.5x^2) \text{ dollars}$$

Since there are two x^2 terms, simplify this expression by collecting these terms together and combining them into one.

$$\begin{aligned}(x^2 + 8x) + (1.5x^2) &= (x^2 + 1.5x^2) + (8x)\\ &= (1 + 1.5)x^2 + 8x\\ &= 2.5x^2 + 8x \text{ dollars}\end{aligned}$$

Notice that this polynomial is shorter than the original, since each power of x occurs only once.

Introduce another example to demonstrate how to add polynomials.

Example 2 **Add $x^3 + x + 1$ and $3x^3 + x^2 + 2x$.**

Solution First, write the polynomials side by side.

$$(x^3 + x + 1) + (3x^3 + x^2 + 2x)$$

The monomials x^3 and x each occur twice, so group the terms together and combine them as we did in Example 1.

$$\begin{aligned}x^3 + x + 1 + 3x^3 + x^2 + 2x &= (x^3 + 3x^3) + x^2 + (x + 2x) + 1\\ &= 4x^3 + x^2 + 3x + 1\end{aligned}$$

Point out that this polynomial is simpler than the original two polynomials written side by side.

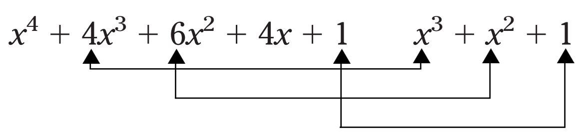
Like Terms

To be explicit about the steps taken to add polynomials, introduce the notion of **like terms**.

Definition of Like Terms	When we are given two polynomials, we say the monomials in the polynomials are like terms if they contain exactly the same number of occurrences of each variable.

Example 3 **Name the like terms in $x^4 + 4x^3 + 6x^2 + 4x + 1$ and $x^3 + x^2 + 1$.**

Solution There are several pairs of like terms, shown in the diagram below.

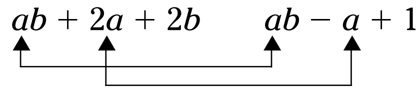


The pairs of like terms are connected by arrows.

So, the like terms are $4x^3$ and x^3, $6x^2$ and x^2, 1 and 1.

Example 4 **Name the like terms in $ab + 2a + 2b$ and $ab - a + 1$.**

Solution In this case, there are two pairs of like terms, ab and ab, $2a$ and $-a$, shown in the diagram below.



Whenever there are like terms, collect them and add them together to get a single term. In Example 3, add $4x^3$ and x^3 together to get $5x^3$. In the same way, add $6x^2$ and x^2 together to get $7x^2$, and finally, $1 + 1 = 2$. When we do this, we get a simpler polynomial.

$$(x^4 + 4x^3 + 6x^2 + 4x + 1) + (x^3 + x^2 + 1) = x^4 + 5x^3 + 7x^2 + 4x + 2$$

In Example 4, collect the ab and a terms to get

$$(ab + 2a + 2b) + (ab - a + 1) = 2ab + a + 2b + 1.$$

Key Idea	Sums of polynomials can be simplified by adding together like terms. In the same way, differences of polynomials can be simplified by collecting together all pairs of like terms.

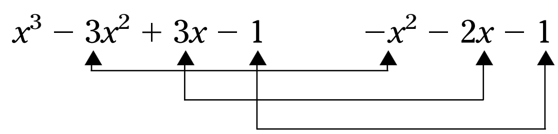
Subtracting Polynomials

Example 5 **Simplify $(x^3 - 3x^2 + 3x - 1) - (x^2 + 2x + 1)$.**

Solution First write this as an addition expression by adding the additive inverse.

$$(x^3 - 3x^2 + 3x - 1) - (x^2 + 2x + 1)$$
$$= (x^3 - 3x^2 + 3x - 1) + (-x^2 - 2x - 1)$$

To simplify the difference, collect all pairs of like terms. These are shown in the diagram below.



Collect the terms to get the following polynomial.

$$x^3 + (-3 - 1)x^2 + (3 - 2)x + (-1 - 1) = x^3 - 4x^2 + x - 2$$

There is another method of adding and subtracting polynomials that is similar to adding and subtracting numbers. Write the polynomials one over the other, with like terms lined up in columns. To find the difference, either subtract the coefficients in each of the columns, or add the additive inverse.

Example 6 **Subtract $4x^3 + 5x^2 + 6x + 7$ and $2x^3 + 3x + 4$.**

Solution Write the polynomials with the like terms in columns.

$$\begin{array}{r} 4x^3 + 5x^2 + 6x + 7 \\ \underline{(-)\ 2x^3 + 0x^2 + 3x + 4} \end{array} \Rightarrow \begin{array}{r} 4x^3 + 5x^2 + 6x + 7 \\ \underline{(+)\ -2x^3 - 0x^2 - 3x - 4} \end{array}$$

Notice that a zero coefficient is added for each missing term. Now add the coefficients in each column.

$$\begin{array}{r} 4x^3 + 5x^2 + 6x + 7 \\ \underline{(+)\ -2x^3 - 0x^2 - 3x - 4} \\ 2x^3 + 5x^2 + 3x + 3 \end{array}$$