Key Concepts

Lesson 26

Multiplying Polynomials

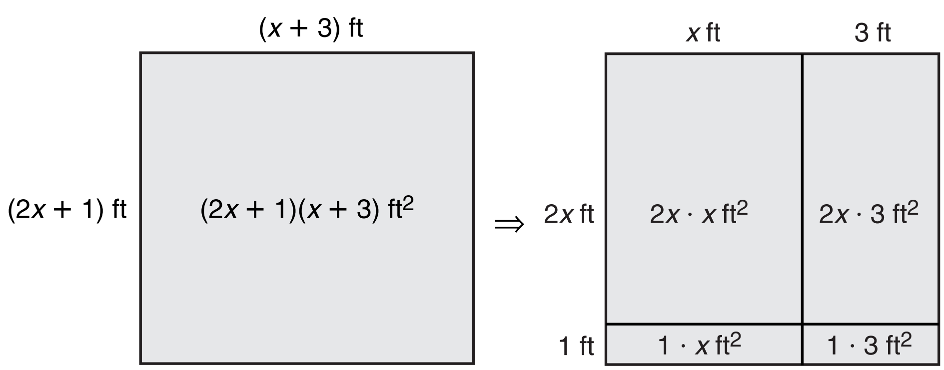
Objective Teach students the techniques of multiplying two binomials by using the FOIL method, and multiplying polynomials by using the Distributive Property.

Note to the Teacher *In this lesson, students will use the technique of multiplying a polynomial by a monomial that they learned in Lesson 9-6, along with the Distributive Property, to multiply any two polynomials. Do Example 1 on the chalkboard to illustrate how the Distributive Property is used when multiplying two binomials.*

Multiplying Binomials

Example 1 **Find the area of a rectangular region of height $(2x + 1)$ feet and width $(x + 3)$ feet.**

Solution Draw a picture of the region. Now draw a picture of the same region, divided as follows.



So, the area of the large rectangle, $(2x + 1)(x + 3)$, is equal to the sum of the areas of the four smaller rectangles.

$$\begin{aligned}(2x + 1)(x + 3) &= (2x \cdot x) + (2x \cdot 3) + (1 \cdot x) + (1 \cdot 3)\\ &= 2x^2 + 6x + x + 3\\ &= 2x^2 + 7x + 3\end{aligned}$$

Notice that in this example the product of the binomials $(2x + 1)$ and $(x + 3)$ is written as the sum of four expressions.

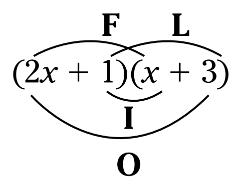
$$(2x + 1)(x + 3) = (2x \cdot x) + (2x \cdot 3) + (1 \cdot x) + (1 \cdot 3)$$

This illustrates a useful shortcut for multiplying binomials. It is called the **FOIL method,** where FOIL stands for First, Outer, Inner, Last.

Write the following description of the FOIL method on the chalkboard.

FOIL Method for Multiplying Two Binomials	To multiply two binomials, find the sum of the products of **F** the First terms, **O** the Outer terms, **I** the Inner terms, and **L** the Last terms.

In Example 1, the FOIL method worked this way:



$$\overset{\text{}}{(2x+1)(x+3)} = \overset{\text{F}}{(2x \cdot x)} + \overset{\text{O}}{(2x \cdot 3)} + \overset{\text{I}}{(1 \cdot x)} + \overset{\text{L}}{(1 \cdot 3)}$$

Now illustrate this method by doing a few examples.

Example 2 **Find $(3x + 2)(2x + 4)$.**

Solution Apply the FOIL method by adding the product of the First, Outer, Inner, and Last terms of the binomials.

$$(3x+2)(2x+4) = \overset{\text{F}}{(3x)(2x)} + \overset{\text{O}}{(3x)(4)} + \overset{\text{I}}{(2)(2x)} + \overset{\text{L}}{(2)(4)}$$

Now multiply and add like terms.

$$= 6x^2 + 12x + 4x + 8$$
$$= 6x^2 + 16x + 8$$

Example 3 **Find $(2x^2 - 1)(4x^3 - 3x)$.**

Solution

$$(2x^2 - 1)(4x^3 - 3x) = \overset{\text{F}}{(2x^2)(4x^3)} + \overset{\text{O}}{(2x^2)(-3x)} + \overset{\text{I}}{(-1)(4x^3)} + \overset{\text{L}}{(-1)(-3x)}$$
$$= 8x^5 - 6x^3 - 4x^3 + 3x$$
$$= 8x^5 - 10x^3 + 3x$$

In multiplying more general polynomials, apply the Distributive Property to write the product as a sum of products of polynomials times monomials, and then use the techniques discussed in Lesson 9-6 to multiply polynomials and monomials. Finally, simplify by adding like terms.

Note to the Teacher *Stress that while the following examples are relatively long and involve several steps, students are already familiar with each of the individual steps. If they work slowly and carefully, they will be successful in solving these types of problems.*

Do the following examples on the chalkboard in a step-by-step manner.

Example 4 **Find $(3x^2 + 2x + 1)(x^3 - x)$.**

Solution First use the Distributive Property to write this product as a sum of products of polynomials times monomials.

$$(3x^2 + 2x + 1)(x^3 - x)$$
$$= (3x^2 + 2x + 1)(x^3) + (3x^2 + 2x + 1)(-x)$$

Then multiply each of the terms (again using the Distributive Property).

$$= [(3x^2)(x^3) + (2x)(x^3) + (1)(x^3)] + [(3x^2)(-x) + (2x)(-x) + (1)(-x)]$$
$$= 3x^5 + 2x^4 + x^3 - 3x^3 - 2x^2 - x$$
$$= 3x^5 + 2x^4 - 2x^3 - 2x^2 - x$$

Example 5 **Find $(a^2 - 2a - 3)(a^2 - 4a - 3)$.**

Solution First use the Distributive Property to write this product as a sum of products of polynomials times monomials.

$$(a^2 - 2a - 3)(a^2 - 4a - 3)$$
$$= (a^2 - 2a - 3)(a^2) + (a^2 - 2a - 3)(-4a) + (a^2 - 2a - 3)(-3)$$

Then use the Distributive Property to multiply each of the terms.

$$= [(a^2)(a^2) + (-2a)(a^2) + (-3)(a^2)] + [(a^2)(-4a) + (-2a)(-4a) + (-3)(-4a)] + [(a^2)(-3) + (-2a)(-3) + (-3)(-3)]$$
$$= (a^4 - 2a^3 - 3a^2) + (-4a^3 + 8a^2 + 12a) + (-3a^2 + 6a + 9)$$

Now combine like terms to get the following.

$$= a^4 + (-2a^3 - 4a^3) + (-3a^2 + 8a^2 - 3a^2) + (12a + 6a) + 9$$
$$= a^4 - 6a^3 + 2a^2 + 18a + 9$$