Key Concepts

Lesson 27

Solving Equations by Factoring

Objective Teach students to solve equations that are in factored form.

Note to the Teacher *This lesson begins with a review of factoring, and then uses factored trinomials to solve equations. Stress to students that this technique (factoring) is very useful in problem solving.*

Factoring Expressions

Begin the lesson with a few factoring problems written on the chalkboard. The following are good examples.

Example 1 **Factor $x^2 - 4x - 21$.**

Solution Since the coefficient of x^2 is 1, factor this expression as $(x + a)(x + b)$ for some values of a and b. Now, $(x + a)$ and $(x + b)$ are multiplied to get $x^2 + (a + b)x + ab$, which is supposed to equal $x^2 - 4x - 21$. Comparing $x^2 + (a + b)x + ab$ and $x^2 - 4x - 21$, we see that the sum $a + b = -4$ and the product $ab = -21$. So, the first step is to find the integral factors of -21, for possible choices of a and b.

$-21 = -21 \cdot 1 \qquad -21 = 21 \cdot (-1)$

$-21 = -3 \cdot 7 \qquad -21 = 3 \cdot (-7)$

Look at the above factor pairs of -21 to see if any of them have a sum equal to -4. If $a = 3$ and $b = -7$, then $a + b = -4$. The factorization is given below.

$$\begin{aligned} x^2 - 4x - 21 &= (x + a)(x + b) \\ &= (x + 3)[(x + (-7)] \qquad a = 3,\ b = -7 \\ &= (x + 3)(x - 7) \end{aligned}$$

Students should multiply $(x + 3)$ and $(x - 7)$ to check that the factorization is correct.

Example 2 **Factor $2x^3 - 3x^2 - 2x$.**

Solution There is a common factor of x in each of the terms. So begin by factoring out x.

$$2x^3 - 3x^2 - 2x = x(2x^2 - 3x - 2)$$

The next step is to factor $2x^2 - 3x - 2$. That is, write this expression as $(ax + b)(cx + d)$ for some values of a, b, c, and d. When we multiply this out we get $acx^2 + (ad + bc)x + bd$, which is supposed to equal $2x^2 - 3x - 2$. So, $ac = 2$. Since 2 only has factors 2 and 1, let $a = 2$ and $c = 1$. We then have to find b and d. Since the coefficient of x, $ad + bc$, is now $2d + b \cdot 1$ or $2d + b$, $2d + b = -3$ and $bd = -2$. The factors of -2 are ± 2 and ± 1, so b and d must be ± 2 or ± 1. On the other hand, $2d + b = -3$. By checking the possibilities, we see that we must have $d = -2$ and $b = 1$. Substitute these values for a, b, c, and d.

$$\begin{aligned} 2x^2 - 3x - 2 &= (ax + b)(cx + d) \\ &= (2x + 1)(x - 2) \quad a = 2,\ b = 1,\ c = 1,\ d = -2 \end{aligned}$$

$$\begin{aligned} \text{So, } 2x^3 - 3x^2 - 2x &= x(2x^2 - 3x - 2) \\ &= x(2x + 1)(x - 2). \end{aligned}$$

Again, students should multiply out $x(2x + 1)(x - 2)$ to check the factoring.

Using Factoring to Solve Equations

How is factoring useful in solving equations? Consider, for example the factored equation

$$(3x - 2)(x + 1) = 0.$$

To solve this, use the following important principle.

Zero Product Property	For all numbers a and b, if $ab = 0$, then $a = 0$, $b = 0$, or both a and b equal 0.

Since $(3x - 2)(x + 1) = 0$, then the Zero Product Property states that either $3x - 2 = 0$, $x + 1 = 0$, or both equal zero.

$$\begin{aligned} 3x - 2 &= 0 \qquad \text{or} \qquad & x + 1 &= 0 \\ 3x &= 2 & x &= -1 \\ x &= \tfrac{2}{3} \end{aligned}$$

So, the solutions of $(3x - 2)(x + 1) = 0$ are $x = \frac{2}{3}$ and $x = -1$. Now apply this property to solve an equation that must be factored first.

Example 3 **Find the solutions of $2x^3 - 2x^2 = 12x$.**

Solution First, write this equation so that it is set equal to zero.

$$2x^3 - 2x^2 - 12x = 0$$

$$x(2x^2 - 2x - 12) = 0 \quad \textit{Since x is a factor of all the terms, factor it out.}$$

$$2x(x^2 - x - 6) = 0 \quad \textit{Since each coefficient is even, factor out a 2.}$$

Now factor $x^2 - x - 6$ as a product $(x + a)(x + b)$ for some values of a and b.

$$\begin{aligned}(x + a)(x + b) &= x^2 + (a + b)x + ab \\ &= x^2 - x - 6\end{aligned}$$

In other words, $a + b = -1$ and $ab = -6$. By looking at the factors of -6, we see that $a = 2$ and $b = -3$ satisfy these requirements. So $x^2 - x - 6 = (x + 2)(x - 3)$. (Make sure students check to see if this is correct by multiplying it out.)

$$2x(x^2 - x - 6) = 0$$

$$2x(x + 2)(x - 3) = 0 \quad \textit{Substitution}$$

Use the Zero Product Property to find the solutions.

$$2x = 0 \qquad x + 2 = 0 \qquad x - 3 = 0$$

$$x = 0 \qquad x = -2 \qquad x = 3$$

So, the solutions of the equation are $x = 0$, $x = -2$, and $x = 3$.

Finally, point out that the solutions of the equation are the roots (zeros) of the function

$$y = 2x^3 - 2x^2 - 12x.$$

Graphically, the roots are where the graph of the function $y = 2x^3 - 2x^2 - 12x$ intersects the x-axis. Have students plot points or use a graphing calculator to graph this function. The roots are at $x = 0$, $x = -2$, and $x = 3$.