Key Concepts

Lesson 28

Graphing Quadratic Functions

Objective Introduce geometric properties of a parabola, the graph of a general quadratic function, including the axis of symmetry and the coordinates of the vertex.

Note to the Teacher *Students will be introduced to the shape of the graph of a quadratic function. In so doing, they will be introduced to the line of symmetry and the vertex. Introduce these concepts with lots of examples. They will also be asked to compute values using quadratic equations.*

Quadratic Functions

Begin by reminding students that a *linear function* is a function that involves a single x term (with a coefficient), and a constant term:

$$y = ax + b.$$

Also remind them that the graph of a linear function is a line.

A **quadratic function** is a function that not only involves an x term and a constant term like a linear function, but it also has an x^2 term. Write the following definition on the chalkboard.

Definition of Quadratic Function	A quadratic function is a function that can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

The graphs of all quadratic functions have the same shape. This shape is called a **parabola.**

Graphing Parabolas

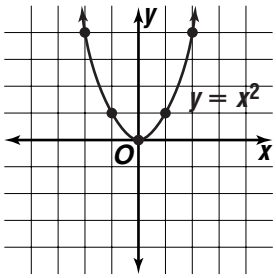
Note to the Teacher *Graph a couple of simple parabolas on the chalkboard by plotting points using a table. Have students also graph a couple by hand. Then graph a few more complicated quadratic functions using a graphing calculator. Again, have your students do a few of these as well.*

Tell your students that the most basic parabola is the graph of the function $y = x^2$.

On the chalkboard, make a table like the one at the right.

x	$y = x^2$	y
-2	$(-2)^2$	4
-1	$(-1)^2$	1
0	$(0)^2$	0
1	$(1)^2$	1
2	$(2)^2$	4

Then plot the points and draw the graph.

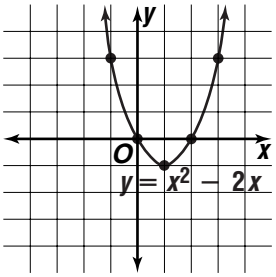


Now plot points and draw the graphs of slightly more complicated functions.

Example 1 **Graph $y = x^2 - 2x$.**

Solution

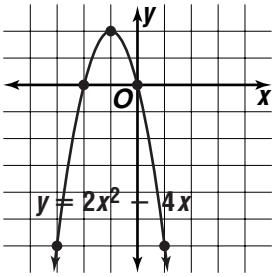
x	$y = x^2 - 2x$	y
-1	$(-1)^2 - 2(-1)$	3
0	$(0)^2 - 2(0)$	0
1	$(1)^2 - 2(1)$	-1
2	$(2)^2 - 2(2)$	0
3	$(3)^2 - 2(3)$	3



Example 2 **Graph $y = -2x^2 - 4x$.**

Solution

x	$y = -2x^2 - 4x$	y
-3	$-2(-3)^2 - 4(-3)$	-6
-2	$-2(-2)^2 - 4(-2)$	0
-1	$-2(-1)^2 - 4(-1)$	2
0	$-2(0)^2 - 4(0)$	0
1	$-2(1)^2 - 4(1)$	-6



Axis of Symmetry

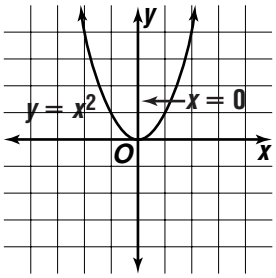
Explain to your students that parabolas are symmetric. Draw one of the parabolas from Example 1 or Example 2 on graph paper. Fold the paper along a vertical line that goes right through the middle of the parabola. This line is called the **axis of symmetry** of the parabola. Point out how the two halves of the parabola match up after you folded along this line. (See the Modeling Mathematics activity on page 612 of the Student Edition.)

Axis of Symmetry	The equation of the axis of symmetry for the graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Tell students that the reason why the axis of symmetry has this as its equation will be explained in a future lesson. Now show the axis of symmetry in each of the previous examples.

Example 3 **Find the equation of the axis of symmetry for the graph of $y = x^2$.**

Solution Write this function as $y = 1x^2 + 0x + 0$. So, $a = 1$, $b = 0$, and $c = 0$.



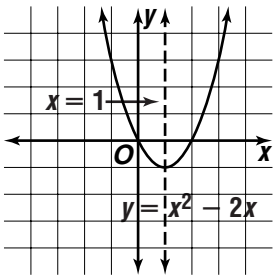
$$x = -\frac{b}{2a}$$

$$= -\frac{0}{2(1)} \text{ or } 0$$

Therefore, the equation of the axis of symmetry is $x = 0$, which is the y-axis.

Example 4 **Find the equation of the axis of symmetry for the graph of $y = x^2 - 2x$.**

Solution Write this function as $y = x^2 - 2x + 0$, so in this case $a = 1$, $b = -2$, and $c = 0$.



$$x = -\frac{b}{2a}$$

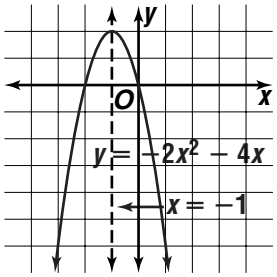
$$= -\frac{-2}{2(1)}$$

$$= \frac{2}{2} \text{ or } 1$$

The equation of the axis of symmetry is $x = 1$.

Example 5 **Find the equation of the axis of symmetry for the graph of $y = -2x^2 - 4x$.**

Solution Write this function as $y = -2x^2 - 4x + 0$ so that in this case $a = -2$, $b = -4$, and $c = 0$.



$$x = -\frac{b}{2a}$$

$$= -\frac{-4}{2(-2)}$$

$$= -\frac{-4}{-4} \text{ or } -1$$

So the equation of the axis of symmetry is $x = -1$.

Vertex of a Parabola

Point out that the axis of symmetry intersects the parabola at exactly one point. This is called the **vertex** of the parabola. This point is either the minimum point on the parabola (as in Examples 3 and 4) or the maximum point (as in Example 5). Graphically, the vertex is the *tip* of the parabola.

Show your students how to compute the coordinates of the vertex. Namely, if we know the equation of the axis of symmetry $\left(x = -\frac{b}{2a}\right)$, then since the vertex lies on the axis of symmetry, we know the x-coordinate of the vertex $\left(-\frac{b}{2a}\right)$. To find the y-coordinate, substitute the x value into the equation of the function.

Example 6 **Find the vertex of the graph of $y = x^2 - 2x$.**

Solution In Example 4, we found that the axis of symmetry of this parabola is $x = 1$. So the vertex has an x-coordinate equal to 1. To find the y-coordinate, substitute $x = 1$ into the equation.

$$y = x^2 - 2x$$
$$y = (1)^2 - 2(1)$$
$$y = -1$$

So the y-coordinate of the vertex is -1. The vertex of this parabola is at $(1, -1)$. This point is the *minimum* since, as the graph shows, the tip of the parabola is at the bottom.

Example 7 **Find the vertex of the graph of $y = -2x^2 - 4x$.**

Solution In Example 5, we found that the axis of symmetry of this parabola is the line $x = -1$. So the vertex has an x-coordinate equal to -1. To find the y-coordinate, substitute $x = -1$ into the equation.

$$y = -2x^2 - 4x$$
$$y = -2(-1)^2 - 4(-1)$$
$$y = 2$$

So 2 is the y-coordinate of the vertex. The vertex of this parabola is at $(-1, 2)$. This point is the *maximum* since, as the graph shows, the tip of this parabola is at the top.

Ask students, "If you are given the coordinates of the vertex, how can you find the axis of symmetry of the parabola?" **Since the vertex lies on the axis of symmetry, which is vertical, the x-coordinate of the vertex will tell you the equation of the axis of symmetry. For example, if the vertex of a parabola is at the point (2, 3), then the axis of symmetry has equation $x = 2$.**