Key Concepts

Graphing Technology: Parent and Family Graphs

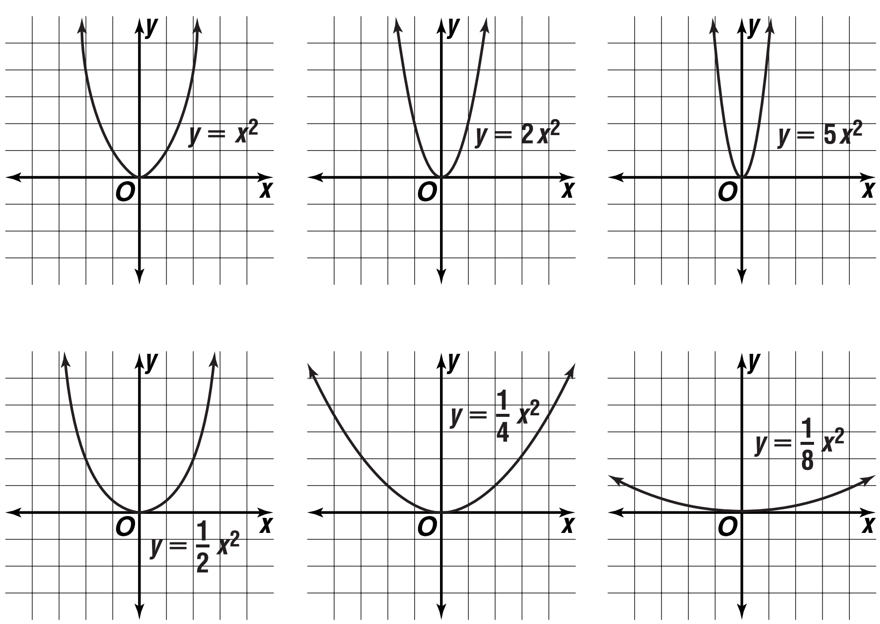
Objective Use a graphing calculator to teach students how the graphs of quadratic functions change as the coefficients change.

Note to the Teacher *In this lesson, start with the parent function* $y = x^2$ *and study how the graphs change in shape or in position when the coefficients are changed. Students should use graphing calculators or computers to graph the families of functions, and then the similarities and differences should be a topic of class discussion.*

Multiplying x^2 by a Positive Number

Have your students graph $y = x^2$ and then the following family of functions.

$$y = 2x^2 \qquad y = 5x^2 \qquad y = \frac{1}{2}x^2 \qquad y = \frac{1}{4}x^2 \qquad y = \frac{1}{8}x^2$$



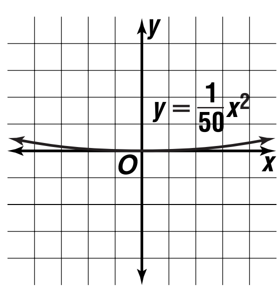
Note to the Teacher *Guide students in a discussion about how the sizes of these parabolas change as the coefficient of x^2 increases or decreases. Make sure they notice the following characteristics.*

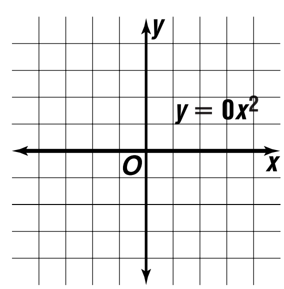
1. The vertices and the axes of symmetry of all these parabolas are the same. Namely, the axis of symmetry is the line $x = 0$, and the coordinates of the vertex are (0, 0). This is because in a quadratic function of the form $y = ax^2 + 0x + 0$, $b = 0$, so the axis of symmetry is given by $x = -\frac{b}{2a}$ or 0, which is the y-axis.
2. The graphs of the quadratic functions $y = ax^2$, where $a > 0$, get more narrow as the value of a increases. The graphs get wider as the value of a decreases.

x	$y = \frac{1}{4}x^2$	$y = x^2$	$y = 5x^2$
0	0	0	0
1	$\frac{1}{4}$	1	5
2	1	4	20
3	$\frac{9}{4}$	9	45
4	4	16	80

Why do parabolas change sizes in this way? The larger the value of a in $y = ax^2$, the faster the y variable increases as x grows. To see this, make a table of values of the functions $y = \frac{1}{4}x^2$, $y = x^2$, and $y = 5x^2$. It is clear from this table that the larger the coefficient of x^2, the faster the y value grows. This results in a steeper incline of the graph, or a "narrower" parabola. Similarly, the smaller the coefficient of x^2, the more slowly the y value grows. This results in a more gradual incline of the graph, or a "wider" parabola.

What happens as the coefficient a in $y = ax^2$ gets smaller and smaller (but remains positive), until it approaches 0? **As a approaches 0, the parabolas get wider and wider until they approach a flat line, namely the graph of $y = 0x$ or 0, which is the x-axis.**

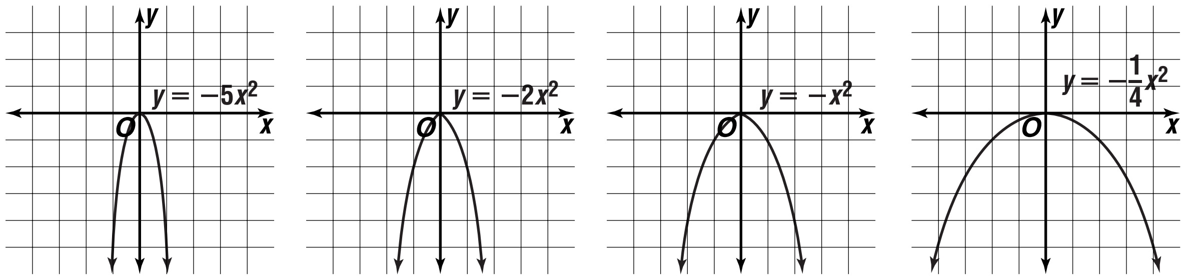




Multiplying x^2 by a Negative Number

Now what happens to the graphs of $y = ax^2$ when a is negative? Have your students graph the following family of functions.

$y = -5x^2 \qquad y = -2x^2 \qquad y = -x^2 \qquad y = -\frac{1}{4}x^2$



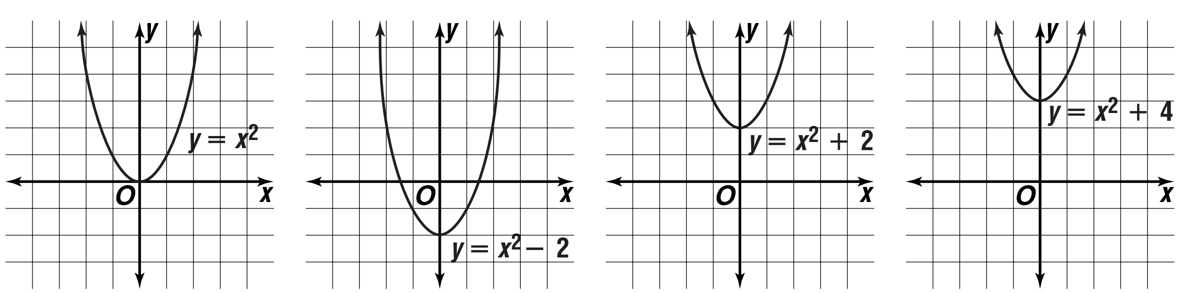
When the a is negative in $y = ax^2$, the parabolas open downwards. This is because as x grows, the y variable becomes more and more negative. So the graphs incline downward. Notice that these graphs have the same characteristics that we saw before.

1. These parabolas all have the same axes of symmetry and vertices. The axis of symmetry is still the vertical line $x = 0$ (or the y-axis), and the vertices are still at (0, 0). This is because in these functions $y = -ax^2 + 0x + 0$, $x = -\frac{b}{2a}$ or 0. Notice however that the vertices are now maximum points of the graphs, where in the earlier examples they were minimum points.
2. Similar to previous examples, as the coefficient a in $y = ax^2$ becomes smaller (more negative), the parabola becomes more narrow. As a gets closer to zero, the parabola gets wider. This is because the smaller the coefficient a is, the faster the y value decreases (gets more negative) as the x value grows. Have your students make tables of values of the functions graphed above to verify this.

Adding a Constant

Now consider a family of quadratic functions that are found by adding a constant to $y = x^2$. Have your students graph the following functions on graphing calculators.

$y = x^2 \qquad y = x^2 - 2 \qquad y = x^2 + 2 \qquad y = x^2 + 4$



Note to the Teacher *Guide a classroom discussion about what happens to the graphs of quadratic functions of the form $y = x^2 + c$ as c changes. Make sure your students notice the following.*

1. The axis of symmetry for each parabola is the same, namely, the vertical line $x = 0$, which is the y-axis. This is because in $y = x^2 + c = 1x^2 + 0x + c$, $a = 1$, and $b = 0$. So, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$ or 0.

2. The vertex of the parabola moves up or down, depending on the value of c in $y = x^2 + c$. Since the axis of symmetry is the line $x = 0$ (or the y-axis), the x-coordinate of the vertex is 0. To find the y-coordinate, substitute $x = 0$ into the equation.

   $$y = x^2 + c$$
   $$y = 0^2 + c \qquad \text{Replace } x \text{ with } 0.$$
   $$y = c$$

   So the y-coordinate of the vertex is c. In other words, the vertex of the parabola given by the function $y = x^2 + c$ is the point at $(0, c)$. So if we add c to $y = x^2$, the vertex moves up by c. If we subtract c from $y = x^2$, the vertex moves down by c.

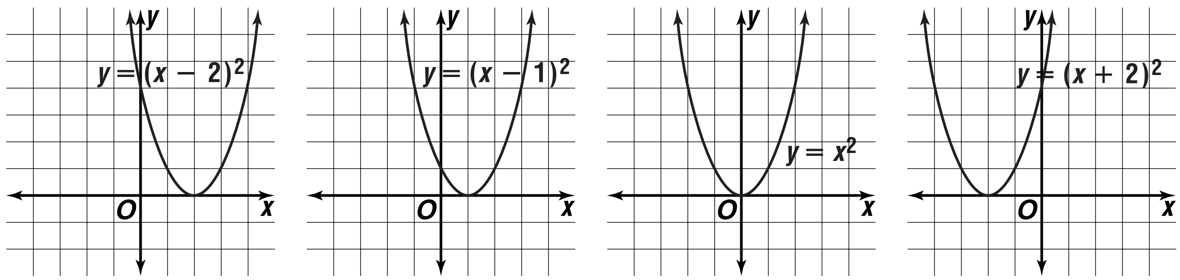
3. The parabolas all have the same size. They have just been shifted up or down by c units because the y values of the function $y = x^2 + c$ are exactly c units more than the y values of the function $y = x^2$. This is called a **vertical translation**. You can see this pattern with a table like the one shown below.

x	$y = x^2$	$y = x^2 + 4$
−1	1	5
0	0	4
1	1	5
2	4	8
3	9	13

Adding a Constant Before Squaring

Now consider the family of quadratic functions of the form $y = (x + d)^2$, where d is some constant. Have your students graph the following examples on graphing calculators.

$y = (x - 2)^2 \qquad y = (x - 1)^2 \qquad y = x^2 \qquad y = (x + 2)^2$



Note to the Teacher *Guide a classroom discussion about what happens to the graphs of quadratic functions of the form $y = (x + d)^2$ as d changes. Make sure your students notice the following.*

1. The axis of symmetry shifts to the left or right, depending on whether d is positive or negative. To understand this algebraically, expand the expression

$$y = (x + d)^2 = x^2 + 2dx + d^2.$$

In this expression, $a = 1$, $b = 2d$, and $c = d^2$. So the axis of symmetry has the equation $x = -\frac{b}{2a} = -\frac{2d}{2} = -d$. For example, if d is positive, the line $x = -d$ is the vertical line shifted to the left d units from the y-axis.

2. The vertex of the parabola moves left or right, depending on the value of d in $y = (x + d)^2$. Since the axis of symmetry is the line $x = -d$, the x-coordinate of the vertex is $-d$. To find the y-coordinate, substitute $x = -d$ into the equation.

$$\begin{aligned} y &= (x + d)^2 \\ &= (-d + d)^2 \quad \textit{Replace } x \textit{ with } -d. \\ &= 0^2 \text{ or } 0 \end{aligned}$$

So the y-coordinate of the vertex is 0. In other words, the vertex of the parabola given by the function $y = (x + d)^2$ is the point at $(-d, 0)$.

3. The parabolas all have the same size. They have just been shifted to the right or left by d units. This is called a **horizontal translation**.