Key Concepts

Lesson 32

Solving Quadratic Equations by Using the Quadratic Formula

Objective Introduce the Quadratic Formula and teach students how to use it to solve quadratic equations.

Note to the Teacher *In this lesson, students use the Quadratic Formula to solve quadratic equations. The goal is for students to become comfortable using this formula. This will require lots of practice, and so the lesson should consist of doing several examples on the chalkboard, and then allowing students to work through problems on their own or in small groups. The Quadratic Formula will be verified in Lesson 13-6 using the technique of completing the square.*

The Quadratic Formula

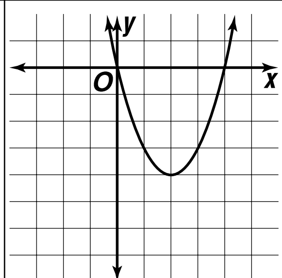
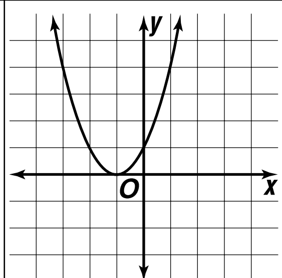
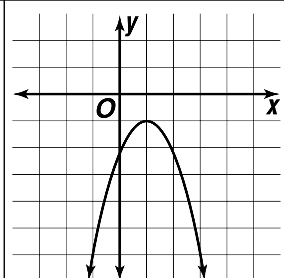
Begin by writing the Quadratic Formula on the chalkboard.

The Quadratic Formula	The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Stress that this formula can be used to find the solutions to *any* quadratic equation.

Notice the $\pm$ sign in the formula. This means that there may be two solutions to a quadratic equation. In Lesson 11-2, we learned that when a parabola intersects the x-axis in two places, the corresponding equation has two solutions. This corresponds to the two solutions $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The expression inside of the square root, $b^2 - 4ac$, is called the **discriminant**. When the discriminant is positive, there are two distinct solutions. When this expression is zero, the square root in the Quadratic Formula is zero, and then there is only one solution, $-\frac{b}{2a}$. This corresponds to the parabola intersecting the x-axis in only one point, namely the vertex. The final case is when the discriminant $b^2 - 4ac$ is negative, in which case the square root $\sqrt{b^2 - 4ac}$ is not defined. In this case, there are no real solutions to the quadratic equation. This corresponds to the parabola not intersecting the x-axis at all.

Solutions of the Quadratic Equation $ax^2 + bx + c = 0$			
Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number of Solutions	2	1	0
Example			
Parabola Intersects the x-axis	yes, in two distinct points	yes, in exactly one point, the vertex	no

Do the following examples on the chalkboard.

Example 1 **Solve $x^2 + x - 2 = 0$ by using the Quadratic Formula.**

Solution In this equation, $a = 1$, $b = 1$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 8}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

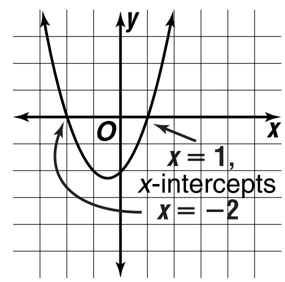
Note that the discriminant $b^2 - 4ac = 9$. Since this is greater than zero, we expect two solutions. Indeed, the Quadratic Formula says that there are two solutions.

$$x = \frac{-1 + 3}{2} \qquad \text{or} \qquad x = \frac{-1 - 3}{2}$$

$$x = \frac{2}{2} \qquad\qquad x = \frac{-4}{2}$$

$$x = 1 \qquad\qquad x = -2$$

Check these solutions by factoring and graphing.



$$x^2 + x - 2 = (x - 1)(x + 2)$$

$$x - 1 = 0 \qquad \text{or} \qquad x + 2 = 0$$

$$x = 1 \qquad\qquad = -2$$

Example 2 **Solve $-x^2 - 2x + 2 = 0$ by using the Quadratic Formula.**

Solution In this equation, $a = -1$, $b = -2$, and $c = 2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(2)}}{2(-1)}$$

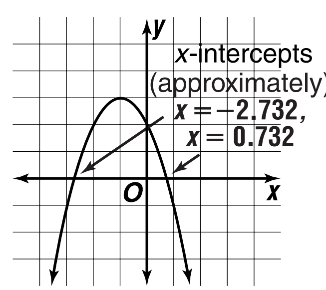
$$= \frac{2 \pm \sqrt{4 - (-8)}}{-2}$$

$$= \frac{2 \pm \sqrt{12}}{-2}$$

$$= \frac{2 \pm 2\sqrt{3}}{-2}$$

$$= -1 \pm -\sqrt{3}$$

So the solutions are $x = -1 - \sqrt{3}$ and $x = -1 + \sqrt{3}$. These numbers are approximately −2.732 and 0.732.



Notice that we could not have solved this equation by factoring, since the function $y = -x^2 - 2x + 2$ does not have integer roots.

End of Lesson