Key Concepts

Lesson 35

Simplifying Rational Expressions

Objective Teach students to find excluded values of a rational expression, and to simplify rational expressions.

Note to the Teacher *Begin this lesson by describing what a rational expression is, and why the domain of the corresponding rational function has to exclude certain values. Then discuss techniques for simplifying rational expressions.*

Excluded Values

Definition of Rational Expressions	A rational expression is an algebraic fraction whose numerator and denominator are polynomials. $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials

The term *rational* comes from the word *ratio*. A rational expression can be described as a ratio of polynomials. A **rational function** is also a ratio of polynomials, as shown below.

$$y = f(x) = \frac{p(x)}{q(x)}$$

Notice that $f(x)$ cannot be defined for values of x when the denominator $q(x)$ equals 0, since division by 0 is undefined. So these values are excluded from the domain of $f(x)$. Do the following example on the chalkboard.

Example 1 **Find the excluded values of $\frac{2x^2 + 8x}{x^2 + 2x - 3}$.**

Solution The excluded values are those values of x where the denominator $x^2 + 2x - 3$ equals 0. Since these are the roots of the denominator, solve $x^2 + 2x - 3 = 0$.

The easiest way to solve this quadratic equation is by factoring and then using the Zero Product Property.

$$x^2 + 2x - 3 = 0$$

$$(x - 1)(x + 3) = 0 \qquad \textit{Factor.}$$

$$x - 1 = 0 \quad \text{or} \quad x + 3 = 0 \qquad \textit{Zero Product Property}$$

$$x = 1 \qquad\qquad x = -3$$

So, the excluded values are $x = 1$ and $x = -3$. Said another way, the *domain* of the rational function $y = \frac{2x^2 + 8x}{x^2 + 2x - 3}$ consists of all values of x except $x = 1$ and $x = -3$.

Example 2 **Find the excluded values of $\frac{x^3y + yz^2}{z(y + 1)x^3}$.**

Solution The excluded values are the zeros of denominator, $z(y + 1)x^3$. Use the Zero Product Property to find these values.

$$z(y + 1)x^3 = 0$$
$$z = 0 \quad \text{or} \quad y + 1 = 0 \quad \text{or} \quad x^3 = 0$$
$$y = -1 \qquad x = 0$$

So, the excluded values are $z = 0$, $y = -1$, and $x = 0$.

Simplifying Rational Expressions

Remind students that to simplify a rational number such as $\frac{6}{10}$, they should look for the greatest common factor (GCF) of the numerator and the denominator, and cancel this common factor.

$$\frac{6}{10} = \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 5} = \frac{3}{5}$$

Simplifying rational expressions can be done the same way. Factor the numerator and the denominator, and cancel the greatest common factor.

Example 3 **Simplify $\frac{x + 2}{x^2 - 4}$.**

Solution First, find the GCF of the numerator and the denominator. The numerator $x + 2$ is completely factored. Factor the denominator.

$$x^2 - 4 = (x - 2)(x + 2)$$

So, the GCF of the numerator and the denominator is $x + 2$. Now simplify the expression by canceling this GCF.

$$\frac{x + 2}{x^2 - 4} = \frac{\overset{1}{\cancel{x + 2}}}{(x - 2) \cdot (\underset{1}{\cancel{x + 2}})}$$
$$= \frac{1}{x - 2}$$

Example 4 **Simplify** $\dfrac{4x^2y(x^2 + 2x + 1)}{2xy^3(x + 1)}$.

Solution First, find the GCF of the numerator and the denominator. Notice that the denominator $2xy^3(x + 1)$ is completely factored. However, the numerator can be factored further.

$$4x^2y(x^2 + 2x + 1) = 2 \cdot 2x^2y(x + 1)^2$$

Comparing the factors of the numerator and the denominator, we see that the greatest common factor of the numerator and denominator is

$$2xy(x + 1).$$

Therefore, simplify the expression by canceling this GCF.

$$\frac{4x^2y(x^2 + 2x + 1)}{2xy^3(x + 1)} = \frac{2x \cdot 2xy(x + 1)(x + 1)}{2xy \cdot y^2(x + 1)}$$

$$= \frac{2x(x + 1) \cdot \overset{1}{\cancel{2xy}}(\overset{1}{\cancel{x + 1}})}{y^2 \cdot \underset{1}{\cancel{2xy}}(\underset{1}{\cancel{x + 1}})}$$

$$= \frac{2x(x + 1)}{y^2}$$