Key Concepts

Lesson 36

Multiplying Rational Expressions

Objective Teach students to multiply rational expressions.

Note to the Teacher *In this lesson, students will learn to multiply rational expressions. The techniques are completely analogous to multiplying rational numbers (fractions). Point out, and continue to stress this analogy throughout the entire lesson.*

Multiplying Rational Numbers

Begin by reminding students that to multiply rational numbers, multiply the numerators and then divide by the product of the denominators. Then simplify by dividing by the common factors of the numerator and the denominator.

Do the following example on the chalkboard.

Example 1 **Find $\frac{4}{9} \cdot \frac{3}{2}$.**

Solution $\frac{4}{9} \cdot \frac{3}{2} = \frac{4 \cdot 3}{9 \cdot 2}$

$= \frac{12}{18}$ *The GCF of the numerator and the denominator is 6.*

$= \frac{\overset{1}{\cancel{6}} \cdot 2}{\underset{1}{\cancel{6}} \cdot 3}$ *Cancel the GCF.*

$= \frac{2}{3}$

An alternative way to approach this problem is to divide by the common factors before multiplying.

$\frac{4}{9} \cdot \frac{3}{2} = \frac{2 \cdot \overset{1}{\cancel{2}}}{3 \cdot \underset{1}{\cancel{3}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}$

$= \frac{2}{3} \cdot \frac{1}{1}$

$= \frac{2}{3}$

Multiplying Rational Expressions

Multiplying rational expressions is done the same way. Multiply the numerators, and then divide the product by the product of the denominators. Then simplify by dividing the common factors of the numerator and the denominator.

Do the following example on the chalkboard.

Example 2 **Find $\frac{2x^2y}{9z^3} \cdot \frac{3xz}{4y^2}$.**

Solution **Method 1:**

$$\frac{2x^2y}{9z^3} \cdot \frac{3xz}{4y^2} = \frac{6x^3yz}{36z^3y^2}$$

$$= \frac{x^3 \cdot \overset{1}{\cancel{6yz}}}{6z^2y \cdot \underset{1}{\cancel{6yz}}}$$ *Cancel the GCF of the numerator and the denominator, 6yz.*

$$= \frac{x^3}{6z^2y}$$

Now just like with multiplying fractions, there is an alternative method. Namely, divide by the common factors before multiplying.

Method 2:

$$\frac{2x^2y}{9z^3} \cdot \frac{3xz}{4y^2} = \frac{x^2 \cdot \overset{1}{\cancel{2y}}}{3z^2 \cdot \underset{1}{\cancel{3z}}} \cdot \frac{x \cdot \overset{1}{\cancel{3z}}}{2y \cdot \underset{1}{\cancel{2y}}}$$

$$= \frac{x^2}{3z^2} \cdot \frac{x}{2y}$$

$$= \frac{x^3}{6z^2y}$$

There are some problems that require factoring the polynomials in order to find the GCF of the numerator and denominator.

Example 3 **Find** $\dfrac{x^2 + 3x + 2}{x - 1} \cdot \dfrac{x^2 - 1}{x + 2}$.

Solution $\dfrac{x^2 + 3x + 2}{x - 1} \cdot \dfrac{x^2 - 1}{x + 2} = \dfrac{(x^2 + 3x + 2)(x^2 - 1)}{(x - 1)(x + 2)}$

To simplify, factor the quadratic expressions in the numerator and then cancel the common factors of the numerator and the denominator.

$$\begin{aligned} \frac{(x^2 + 3x + 2)(x^2 - 1)}{(x - 1)(x + 2)} &= \frac{(x + 1)\overset{1}{\cancel{(x + 2)}}\overset{1}{\cancel{(x - 1)}}(x + 1)}{\underset{1}{\cancel{(x - 1)}}\underset{1}{\cancel{(x + 2)}}} \\ &= \frac{(x + 1)(x + 1)}{1} \\ &= (x + 1)^2 \\ &= x^2 + 2x + 1 \end{aligned}$$

For practice, do one more example.

Example 4 **Find** $(x - 3)\dfrac{x + 2}{x^2 - 4x + 3}$.

Solution $(x - 3)\dfrac{x + 2}{x^2 - 4x + 3} = \dfrac{(x - 3)(x + 2)}{x^2 - 4x + 3}$

First, find the common factors. To do this, factor the quadratic expression in the denominator.

$$\begin{aligned} \frac{(x - 3)(x + 2)}{x^2 - 4x + 3} &= \frac{\overset{1}{\cancel{(x - 3)}}\,(x + 2)}{\underset{1}{\cancel{(x - 3)}}\,(x - 1)} \quad \textit{Cancel the common factors.} \\ &= \frac{x + 2}{x - 1} \end{aligned}$$

We therefore conclude that

$$(x - 3)\frac{x + 2}{x^2 - 4x + 3} = \frac{x + 2}{x - 1}.$$

Note to the Teacher *This technique is very important because it reinforces prior techniques like factoring and multiplication of rational numbers. Give your students many problems to practice.*