# Key Concepts

Lesson 37

## Dividing Rational Expressions

**Objective** Teach students to divide rational expressions.

**Note to the Teacher** *In this lesson, students will learn to divide rational expressions. The techniques are completely analogous to dividing rational numbers (fractions). Point out, and continue to stress this analogy throughout the entire lesson.*

### Dividing Rational Numbers

Begin by reminding students that to divide rational numbers, use the "invert and multiply" rule. That is, invert the divisor (take the reciprocal), and then multiply by the numerator.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Then simplify by dividing by the common factors of the numerator and the denominator.

Do the following example on the chalkboard.

**Example 1** **Find $\frac{4}{15} \div \frac{2}{3}$.**

**Solution** Take the reciprocal of the divisor $\left(\frac{2}{3}\right)$ and multiply.

$$\frac{4}{15} \div \frac{2}{3} = \frac{4}{15} \cdot \frac{3}{2}$$

Before multiplying, look for common factors.

$$\frac{4}{15} \cdot \frac{3}{2} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{3}} \cdot 5} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{2}{5} \cdot \frac{1}{1} = \frac{2}{5}$$

So, we conclude that

$$\frac{4}{15} \div \frac{2}{3} = \frac{2}{5}.$$

## Dividing Rational Expressions

Dividing rational expressions is done the same way. Take the reciprocal of (invert) the divisor and then multiply by the numerator. Then simplify by dividing the common factors of the numerator and the denominator.

Do the following example on the chalkboard.

**Example 2** **Find $\frac{3xy^2}{4z^3} \div \frac{9x^2y}{2z^2}$.**

**Solution** Take the reciprocal of the divisor and multiply.

$$\frac{3xy^2}{4z^3} \div \frac{9x^2y}{2z^2} = \frac{3xy^2}{4z^3} \cdot \frac{2z^2}{9x^2y}$$

Before multiplying, look for common factors.

$$\frac{3xy^2}{4z^3} \cdot \frac{2z^2}{9x^2y} = \frac{y \cdot \overset{1}{\cancel{3xy}}}{2z \cdot \underset{1}{\cancel{2z^2}}} \cdot \frac{1 \cdot \overset{1}{\cancel{2z^2}}}{3x \cdot \underset{1}{\cancel{3xy}}} \quad \textit{Cancel the common factors.}$$

$$= \frac{y}{2z} \cdot \frac{1}{3x}$$

Now multiply these rational expressions.

$$\frac{y}{2z} \cdot \frac{1}{3x} = \frac{y \cdot 1}{2z \cdot 3x} = \frac{y}{6zx}$$

So we conclude that

$$\frac{3xy^2}{4z^3} \div \frac{9x^2y}{2z^2} = \frac{y}{6zx}.$$

There are some problems involving division of rational expressions that require factoring of polynomials in order to find common factors of the numerators and the denominators.

**Example 3** **Find $\frac{x^2 + 6x + 8}{x^2 - x} \div \frac{x^2 + 3x - 4}{x}$.**

**Solution** Take the reciprocal of the divisor and multiply.

$$\frac{x^2 + 6x + 8}{x^2 - x} \div \frac{x^2 + 3x - 4}{x} = \frac{x^2 + 6x + 8}{x^2 - x} \cdot \frac{x}{x^2 + 3x - 4}$$

Before multiplying, factor the quadratic terms in these quadratic expressions in order to find common factors.

$$\frac{x^2 + 6x + 8}{x^2 - x} \cdot \frac{x}{x^2 + 3x - 4} = \frac{(x + 2)\overset{1}{\cancel{(x + 4)}}}{(x - 1)\underset{1}{\cancel{x}}} \cdot \frac{1 \cdot \overset{1}{\cancel{x}}}{(x - 1)\underset{1}{\cancel{(x + 4)}}}$$

$$= \frac{x + 2}{x - 1} \cdot \frac{1}{x - 1}$$

Now multiply these rational expressions.

$$\frac{x + 2}{x - 1} \cdot \frac{1}{x - 1} = \frac{(x + 2) \cdot 1}{(x - 1)(x - 1)}$$

$$= \frac{x + 2}{(x - 1)^2}$$

$$= \frac{x + 2}{x^2 - 2x + 1}$$

So, we conclude that

$$\frac{x^2 + 6x + 8}{x^2 - x} \div \frac{x^2 + 3x - 4}{x} = \frac{x + 2}{x^2 - 2x + 1}.$$

**Note to the Teacher** *This technique is very important because it reinforces prior techniques like multiplying rational expressions, factoring quadratic expressions, and dividing rational numbers. Give your students many practice problems.*