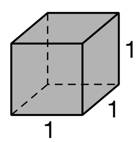
Key Concepts

Lesson 7-5

Volume of Prisms and Cylinders

Objective Teach students the concept of the volume of a 3-dimensional object, and also the formula for the volume of prisms and cylinders.

Note to the Teacher *Volume may be a new concept for your students. Begin with a classroom discussion about the meaning of volume. Guide the discussion so that students talk about the idea of how much liquid a 3-dimensional object would hold, or how much air a balloon shaped like the object would hold. In order to make this idea more precise, stress that the volume of an object is the measure of the space it occupies. Then talk about the basic unit of volume, which is a cube of side length 1 unit. The volume of such a cube is 1 cubic unit. Refer to this cube as a unit cube.*



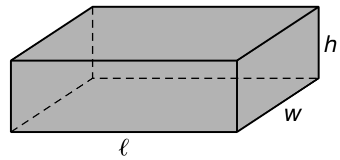
The volume of a cube of side length 1 unit (i.e. 1 inch) is 1 cubic unit (i.e. 1 in^3).

Explain to your students that when we compute the volume of a rectangular prism, we are basically counting how many cubes of side length one unit can fit inside the solid. This is analogous to computing the area of a region by counting how many squares of side length one unit can fit inside that region.

Rectangular Prisms

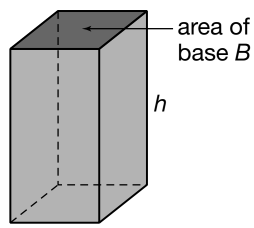
A rectangular prism has three important dimensions, its length, width, and height. Stress that if a rectangular prism is 10 units long, then 10 of the unit cubes can be lined up along its length. Similarly, if its height is 5 units then 5 unit cubes can be stacked up along its height, and if its width is 7 units then 7 unit cubes can be lined up along its width. Help students visualize such a prism completely filled with unit cubes. Point out that there would be 5 layers of unit cubes, each of which would have 10×7 or 70 unit cubes in it. Help them see that this means there would be 5×70 or 350 unit cubes inside the prism.

In general, a rectangular prism of length ℓ, width w, and height h can be filled by $\ell \cdot w \cdot h$ unit cubes. That is, its volume V is given by the product ℓwh.



volume of a rectangular prism = ℓwh

More generally, suppose we have a solid whose height is h. We cut the solid into slices by cutting parallel to the base. Then the top of each slice is the same size and shape as the base, and the volume of such a solid is the area of the base times the height. Using B to represent the area of the base, the volume of the solid is then given by the formula $V = Bh$.

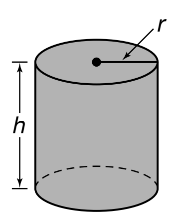


$V = Bh$

In the figure above, the base is shown as a rectangle to show that B replaces ℓw in the formula from the previous lesson. However, point out that the base could be some other shape, such as a triangle, a hexagon, or a circle.

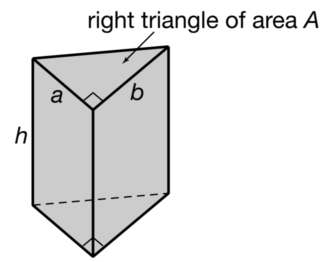
Cylinders

A **cylinder** is a solid whose bases are two congruent circles that are parallel to each other. So in the volume formula $V = Bh$, the base area B is that of a circle. Remind students that the area of a circle is given by the expression πr^2. So the volume of a cylinder with radius r and height h is the area of the circular base, πr^2, times the height h: $V = \pi r^2 h$.

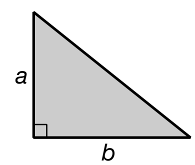


Right Triangular Prisms

In the triangular prism below, the cross-sectional slices parallel to the bases are right triangles of area A. So the volume of a triangular prism of height h is given by $A \cdot h$.



Recall that the area of a right triangle (the base in this right triangular prism) whose leg lengths are a and b as shown is $A = \frac{1}{2}ab$.



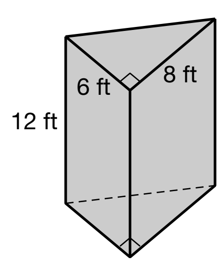
area $A = \frac{1}{2}ab$

Thus the volume of the right triangular prism shown above is given by the formula

$$\begin{aligned} V &= A \cdot h \\ &= \frac{1}{2}abh. \end{aligned}$$

End the lesson by doing some examples in class and assigning others for individual or group work. Here are two good examples.

Example 1 **Find the volume of the right triangular prism shown below.**



Solution We saw that the volume of a right triangular prism is the area of the triangular base times the height. In this case, the area of the right triangle is given by the following.

$A = \frac{1}{2}ab$

$= \frac{1}{2} \cdot 6 \cdot 8$ *Replace a with 6 and b with 8.*

$= 24$

So the volume is as follows.

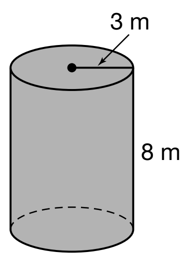
$V = A \cdot h$

$= 24 \cdot 12$ *Replace A with 24 and h with 12.*

$= 288$

The volume of the right triangular prism is 288 cubic feet.

Example 2 **Find the volume of the cylinder shown below.**



Solution Use the formula for the volume of a cylinder, $V = \pi r^2 h$.

$V = \pi r^2 h$

$= \pi(3^2)(8)$ *Replace r with 3 and h with 8.*

$= \pi(9)(8)$

$= 72\pi$

So, the volume of the cylinder is 72π cubic meters. Using a calculator and the approximation 3.14 for π, the volume is about 226.08 cubic meters.