

Key Concepts

Lesson
9-4

Measures of Central Tendency

Objective Teach students to find the mean, median, and mode of a set of data.

Note to the Teacher *The quantities called the mean, median, and mode are referred to as **measures of central tendency** because they are single values that are representative of an entire collection of numbers. That is, each of these values is a number toward which the entire collection of numbers tends.*

Collections of Numbers

We are often confronted with large collections of numbers on which we need to base a decision. Here are two such situations.

- Myra and Ahmed will be traveling to the island of Hawaii in March, and they are trying to decide whether or not to take jackets with them. The collection shown below is the lowest recorded temperature, in degrees Fahrenheit, during the month of March on Hawaii for each of the last 10 years.

$\{68^\circ, 70^\circ, 65^\circ, 72^\circ, 63^\circ, 74^\circ, 70^\circ, 69^\circ, 68^\circ, 64^\circ\}$

If they decide that these temperatures are high enough, Myra and Ahmed will decide that they do not need to take jackets on their trip. On the other hand, if they feel that these low temperatures are too low, they will take jackets on the trip. Myra and Ahmed want to find some way to make a judgment about the entire set of data.

- A grocer is deciding how many pounds of bananas to buy for her store for next week. She has searched through her records for the number of pounds of bananas sold at her store for each of the last 8 weeks. The data are shown below.

$\{110, 115, 98, 136, 114, 105, 124, 131\}$

The grocer needs to estimate how many pounds of bananas she expects to sell during the next week in order to buy an appropriate amount. If she buys too many pounds of bananas, she will have spent too much money because some of the bananas will spoil. On the other hand, if she buys too few pounds of bananas, the store will lose sales because there are not enough bananas to fill customers' needs.

Have a class discussion about how people make decisions like these based on collections of numbers. Conclude the discussion by leading students toward the use of a measure of central tendency as a good indicator of all the data in a collection.

Measures of Central Tendency

In both situations presented on the previous page, it would be helpful to the people involved to be able to summarize the data by representing it with a single number. They could then use that number to make informed decisions. In the case of the Hawaii trip, Myra and Ahmed want to predict the temperature when they will be on the island of Hawaii, and use that prediction to decide about taking jackets on the trip. The grocer wants to predict how many bananas will be sold at her store next week, and use that prediction to make the best banana-purchasing decision possible.

Point out that in this lesson we will be discussing three different numbers that are representative of a collection of numbers. The first of these representative numbers is the mean.

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| Mean | The mean of a collection of numbers is the number obtained by adding all the numbers together and then dividing the sum by the number of members in the collection. |
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Your students may have heard the mean referred to as the *average* of a set of data.

Example 1 Refer students to the Hawaii temperature situation presented earlier. Find the mean of the given temperatures. Use the mean to help Myra and Ahmed make a decision about bringing jackets on their trip.

Solution We first find the sum of the temperatures.

$$68 + 70 + 65 + 72 + 63 + 74 + 70 + 69 + 68 + 64 = 683$$

Then we divide the sum by the total number of temperatures in the collection, which is 10.

$$\frac{\text{sum}}{10} \rightarrow \frac{683}{10} = 68.3$$

Using the mean, we can predict that the low temperature on any March day on the island of Hawaii will be about 68.3°F. Myra and Ahmed should not need jackets on their trip.

Example 2 Refer to the grocer's situation about ordering bananas. Find the mean of the given data. Use the mean to suggest a decision about the number of pounds of bananas the grocer should order for next week.

Solution Find the sum of the numbers in the data set.

$$110 + 115 + 98 + 136 + 114 + 105 + 124 + 131 = 933$$

There are eight numbers in the data set, so divide the sum by 8.

$$\frac{\text{sum}}{8} \rightarrow \frac{933}{8} = 116.625$$

The mean indicates that the grocer can expect to sell about 117 pounds of bananas next week, and she should buy approximately that many pounds of bananas.

Note to the Teacher *This is a good time to have the students work some examples on their own, either individually or in small groups. Have your students write their own collections of numbers and compute the means. Have them save their collections so they can compute the medians and modes of these collections.*

A second number we can use to represent a collection of numbers is the median.

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| Median | The median of a collection of numbers is the middle number in the collection when the numbers are listed in order. When there is an odd number of members in a collection, the median can be identified visually. However, when there is an even number of members in a collection, the median is the mean of the two numbers in the middle of the ordered collection. |
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The two possibilities for the computation of the median are shown in the following examples.

Example 3 Find the median of the collection {3, 1, 5, 9, 10, 8, 14}.

Solution To find the median, we must first write the numbers in order. Ordered from least to greatest, the collection is

$$\{1, 3, 5, 8, 9, 10, 14\}.$$

There are seven numbers in the collection, so the median is the fourth number from either end of the list. The median of this collection is 8.

Example 4 Find the median of the collection {4, 2, 7, 1, 9, 6}.

Solution First, we write the numbers in order. → {1, 2, 4, 6, 7, 9}

Since there are an even number of members in this collection, we take the mean of the two middle numbers, which are 4 and 6. So, the median is

$$\frac{4 + 6}{2} = \frac{10}{2} \text{ or } 5.$$

Note to the Teacher *Now have the students compute the medians of the collections they wrote earlier. Have them compare the median with the mean for each of the collections. Students should see that the mean and median for a collection are usually different values.*

The third number we can use to represent a collection of numbers is the mode.

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| Mode | The mode of a collection of numbers is the number (or numbers) that occurs most often in the collection. |
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Note to the Teacher *Explain that a collection of numbers may fail to have a mode, if there is no member that appears more frequently than any other. For example, if each member occurs only once, there is no mode. On the other hand, a collection may have more than one mode. For example, if two of the members of a collection occur twice and all the other members occur just once, then there are two modes.*

Example 5 Identify the mode of the collection

{1, 4, 3, 5, 3, 6, 8, 3, 4, 9}.

Solution We count the number of occurrences of each number, and find that 1, 5, 6, 8, and 9 each occur once, 4 occurs twice, and 3 occurs three times. This means that 3 occurs most often, and we say that the mode is 3.

Example 6 Identify the mode of the collection {1, 1, 2, 3, 4, 5, 5, 6, 7}.

Solution Counting the occurrences of each number: 2, 3, 4, 6, and 7 each occur once, and 1 and 5 each occur twice. Since 1 and 5 both occur more often than the other members of the collection, there are two modes: 1 and 5.

Note to the Teacher *Conclude the lesson by asking the class to find the mean, median, and mode in a particular example. A good example is provided on the next page.*

Example 7 The table at the right shows the number of hits made by the members of the Jets Little League baseball team last season. Find the mean, median, and mode of this collection of numbers.

| Name | Number of Hits |
|---------|----------------|
| Joe | 12 |
| Melissa | 8 |
| Bill | 7 |
| Bob | 14 |
| Susan | 10 |
| Karen | 9 |
| Rick | 10 |
| Julie | 11 |
| Eric | 10 |
| Debbie | 8 |
| Chris | 9 |
| Jessica | 11 |

Solution mean = $\frac{12 + 8 + 7 + 14 + 10 + 9 + 10 + 11 + 10 + 8 + 9 + 11}{12}$
 $= \frac{119}{12}$ or 9.91666...

To compute the median, we rewrite the numbers in increasing order.

$$\{7, 8, 8, 9, 9, 10, 10, 10, 11, 11, 12, 14\}$$

There are two middle numbers, 10 and 10. So the median is 10.

To identify the mode, we see that 7, 12, and 14 each occur once, while 8, 9, and 11 each occur twice, and 10 occurs three times. So, 10 is the mode.

Summarizing, the mean is about 9.9 and the median and mode are both 10.

