Key Concepts

Lesson 10-2

Solving Two-Step Equations

Objective Teach students to solve linear equations that require the use of two operations.

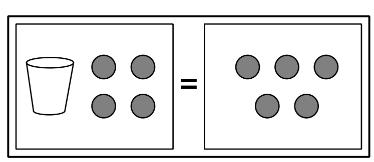
Note to the Teacher *The content of this lesson is crucial in the study of algebra. The more automatic the skills developed in this lesson can be made, the easier will be students' introduction to algebra. Be sure to provide ample opportunity for students to practice these skills. It is often helpful to illustrate the algebra skills using models. However, it is important that students learn to solve the equations without the use of models. While modeling the solution steps for an equation, be sure to finish the discussion by showing the algebra involved in each step.*

Representing Equations with Models

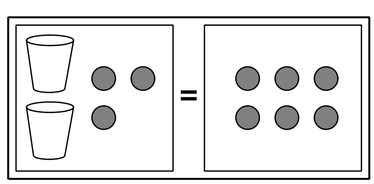
Equations involving a single variable and integers can often be represented using models. The variable is represented by a single object (a cup is used in the Student Edition and here) and counters are used to represent the integers. The counters are shown here as either gray circles for a positive integer or black circles for a negative integer. (In the Student Edition, the counters are either yellow, denoting a positive integer, or red, denoting a negative integer.)

To represent equations here using the models, we draw the same number of cups as the number of times the variable occurs, and place the appropriate number of counters on each side of the equal sign to model the integers occurring in the equation. Here are some examples of these equation models.

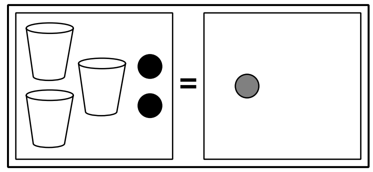
Example 1 $x + 4 = 5$



Example 2 $2x + 3 = 6$



Example 3 $3x - 2 = 1$

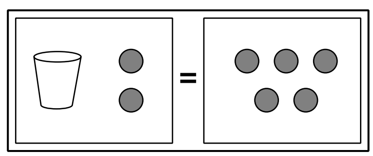


Solving One-Step Equations Using Models

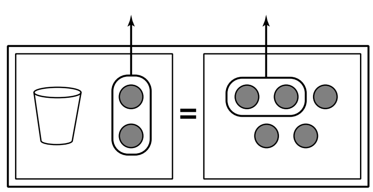
Stress that once the equations have been modeled, they can be manipulated by adding and removing gray or black counters to either side of the model. Also point out that when there are both gray and black counters on one side, the counters can be paired up and removed. The removal of these pairs corresponds the fact that $1 + (-1) = 0$, since a gray counter represents 1 and a black counter represents -1.

Example 4 **Solve $x + 2 = 5$ for x.**

Solution First we model the equation.

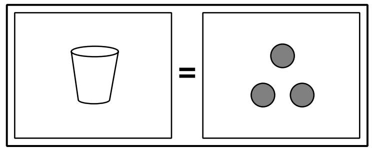


Then we remove the same number of counters from each side until the cup is alone. Stress that it is critical that the same number of counters are removed from each side. To get the cup alone, we need to remove 2 gray counters from each side.



Note to the Teacher *Explain to your students how this manipulation corresponds to subtracting the number 2 from each side of the equation.*

After removing counters (2 from each side), the model now looks like this.

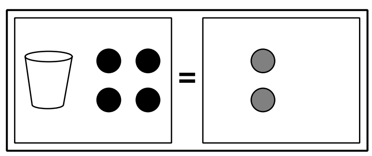


The model above tells us that $x = 3$.

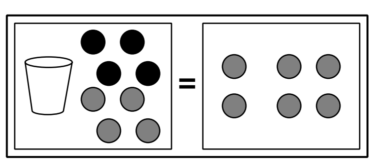
We can verify that this value of x is the correct solution by replacing x with it in the original equation: $3 + 2 = 5$. This is correct.

Example 5 **Use a model to solve $x - 4 = 2$.**

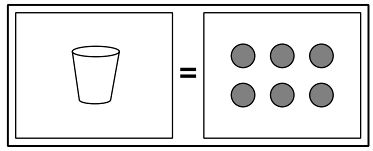
Solution Point out that this equation involves subtraction on one side. Remind students that subtraction of an integer is the same as the addition of the opposite integer. Therefore, the equation $x - 4 = 2$ can be rewritten as the equation $x + (-4) = 2$. The model for this equation is given below.



To get the cup alone in the model, we need to remove the 4 black counters that are on the left side of the equation. But since there are no black counters on the other side, we cannot use the technique shown in Example 4. In order to be able to remove the black counters, we need to add 4 gray counters to each side. As long as we add the same number and color of counters to each side, the new model will be equivalent to the original model. The model with 4 gray counters added to each side is shown below.



Now we can remove the four pairs of gray and black counters from the left side of the model. We are left with this model.

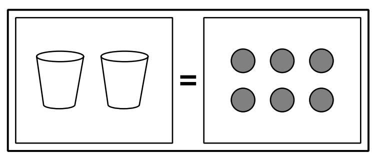


This model represents the equation $x = 6$. Replacing x in the original equation with 6 gives the true number sentence $6 - 4 = 2$, so the solution is correct.

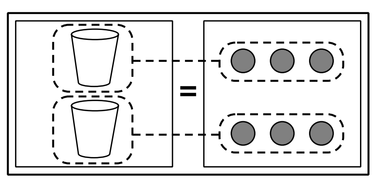
The following equation requires the use of more than one cup in the model. Emphasize to students that the solution of the equation is the value of x, that is, the value of just one x.

Example 6 **Solve $2x = 6$.**

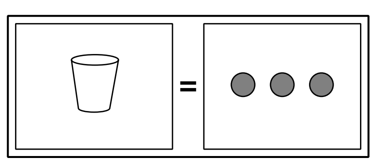
Solution Model the equation.



Point out that thinking of the equation as $x + x = 6$ is helpful at this step. Emphasize that there are two xs modeled on the left side of the figure, and that we can separate the counters on the right side into two groups of 3 counters each. We can then assign one group of 3 counters to each cup.



Since we are interested only in how many counters go with one x, we can focus on just one of the two parts of the previous model.



The model at the bottom of the previous page shows that $x = 3$. This result can be verified by substituting 3 for x in the original equation.

Note to the Teacher *The process in this example corresponds to dividing each side of the equation by 2. Be sure to point this out to the class, since this process is not as clear from the models as it is when we add or remove counters from each side.*

Note to the Teacher *During the remainder of the lesson, feel free to use models for solving the equations. Make sure though that you explain the actual algebraic process as well.*

Solving One-Step Equations Using Algebra

Now is a good time to review the algebraic techniques for solving one-step equations that students learned in Lessons 1-8 and 1-9.

Example 7 **Solve $x + 7 = 15$.**

Solution Subtract 7 from each side of the equation.

$$\begin{array}{rcr} x + 7 & = & 15 \\ -\ 7 & & -\ 7 \\ \hline x + 0 & = & 8 \end{array}$$

So, the solution is 8.

In the example above, we used the Subtraction Property of Equality, which we can think of as *undoing* the addition of 7 on the left side of the equation. It can also be thought of as using the Addition Property of Equality, with a negative number being added to each side.

Example 8 **Solve $3x = 27$.**

Solution Divide each side of the equation by 3.

$$3x = 27$$

$$\frac{3x}{3} = \frac{27}{3}$$

$$x = 9$$

The solution is 9.

In this example, we used the Division Property of Equality, which can be thought of as *undoing* the multiplication by 3 on the left side of the equation.

Solving Two-Step Equations Algebraically

Point out that many real-world situations are modeled by equations that involve both addition and multiplication. Point out that we must use both of the steps shown in the previous two examples to solve these equations.

Example 9 **Martin went to the store and bought a book for $10 and some CDs. The CDs cost $12 each. If Martin spent a total of $46, how many CDs did he buy?**

Solution We first need to write an equation to model the situation. Let C represent the number of CDs Martin bought. Then the amount of money (in dollars) he spent on CDs is

$$12 \cdot C \text{ (or } 12C\text{)},$$

since each CD cost $12.

Since Martin also spent $10 on a book, the total amount of money (in dollars) he spent at the store is

$$12C + 10.$$

The problem says that Martin spent a total of $46, so we can write the equation

$$12C + 10 = 46.$$

To solve this equation, the goal is to have C alone on one side of the equation. This means we must undo the addition by 10 and the multiplication by 12 that occur on the left side. We first eliminate the 10 on the left side by subtracting 10 from each side.

$$\begin{array}{rcr} 12C + 10 & = & 46 \\ -\ 10 & & -\ 10 \\ \hline 12C + \ \ 0 & = & 36 \end{array}$$

So the result of the first step is the equation $12C = 36$. This equation can now be solved by dividing each side by 12.

$$12C = 36$$
$$\frac{12C}{12} = \frac{36}{12}$$
$$C = 3$$

So, Martin bought 3 CDs.

Note to the Teacher *At this point, emphasize to the class the steps that were used to solve the two-step equation in Example 9.*

(1) Add to or subtract from each side to undo the subtraction or addition on one side of the equation. This will leave an equation involving only multiplication or division.

(2) Solve the resulting equation by dividing or multiplying each side of the equation to undo the multiplication or division.

Now do several more examples, pointing out how the two steps are applied each time.

Example 10 **Solve $-2x + 5 = 17$.**

Solution **Step 1** Subtract 5 from each side.

$$\begin{array}{rcr} -2x + 5 & = & 17 \\ -5 & & -5 \\ \hline -2x + 0 & = & 12 \end{array}$$

Step 2 Divide each side by −2.

$$-2x = 12$$
$$\frac{-2x}{-2} = \frac{12}{-2}$$
$$x = -6$$

Example 11 **Solve $\frac{y}{4} + 8 = 12$.**

Solution **Step 1** Subtract 8 from each side.

$$\begin{array}{rcr} \frac{y}{4} + 8 & = & 12 \\ -8 & & -8 \\ \hline \frac{y}{4} + 0 & = & 4 \end{array}$$

Step 2 Multiply each side by 4.

$$4\left(\frac{y}{4}\right) = 4(4)$$
$$y = 16$$

Example 12 **Solve $\frac{3z}{2} - 5 = 4$.**

Solution **Step 1** Add 5 to each side.

$$\begin{array}{rcr} \frac{3z}{2} - 5 & = & 4 \\ + 5 & & + 5 \\ \hline \frac{3z}{2} + 0 & = & 9 \end{array}$$

Step 2 Since $\frac{3z}{2}$ can be rewritten as $\frac{3}{2}z$, multiply each side of the equation by $\frac{2}{3}$, the reciprocal of $\frac{3}{2}$.

$$\frac{2}{3}\left(\frac{3z}{2}\right) = \frac{2}{3}(9)$$

$$z = 6$$

Note to the Teacher *Now ask students to solve several equations for themselves. Practicing these algebra skills is very important. Be sure to include some equations with negative coefficients, and some involving fractions. Several exercises are given below.*

Exercises

Solve each equation.

1. $3x - 7 = 14$ **7**

2. $-7x + 5 = -16$ **3**

3. $-5x + 12 = -3$ **3**

4. $\frac{3t}{5} - 6 = 6$ **20**

5. $1.5n - 3 = 9$ **8**

6. $\frac{w}{4} + 10 = -14$ **−96**