Key Concepts

Lesson 12-7

Multiplying Monomials and Polynomials

Objective Introduce students to the algebra of polynomials, teaching them to use the laws of exponents to multiply monomials, and teach them to multiply monomials and polynomials.

Note to the Teacher *In this lesson we talk in general about multiplying monomials. Students need to understand how to multiply monomials first in order to understand how to multiply a monomial and a polynomial.*

Monomials

Begin by reminding your students about the meaning of an exponent. Point out that x^2 denotes $x \cdot x$, x^3 denotes $x \cdot x \cdot x$, and in general, x^n denotes

$$\underbrace{x \cdot x \cdot x \cdot \cdots \cdot x.}_{n \text{ factors}}$$

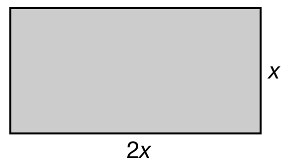
A **monomial** is an expression that is a product of variables and numbers. The numbers are called **constants**. When we have several occurrences of the same variable, we will typically write the expression using an exponent.

- The expressions $5x^2$, $7xy$, $-3x^3$, $4y$, and z^{100} are all monomials. In each case, we have the product of a constant and one or more variables.
- The expressions $3x + 1$, $\frac{1}{x^2 + 2}$, and $\sqrt{x + 3}$ are *not* monomials.

Monomials frequently occur as a result of the mathematical modeling of practical problems.

Example 1 **Suppose we know that the longer side of a rectangle is twice the length of the shorter side. Write an expression for the area of the rectangle.**

Solution Let x represent the length of the shorter side of the rectangle. Then the length of the longer side can be represented as $2x$.



The area of a rectangle is the product of its length and width, so the area of the rectangle can be given by the expression

$$2x \cdot x = 2x^2.$$

So the area of the rectangle is expressed as $2x^2$.

Laws of Exponents and Multiplying Monomials

When multiplying monomials, we often must find the product of two different powers of the same variable, for instance, $x^2 \cdot x^3$. A good way to illustrate this is to show the product of various powers of 2.

Example 2 $\mathbf{2^2 \cdot 2^3 =} (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$
$= 2^5$

Have students verify this result by showing that $2^2 \cdot 2^3 = 4 \cdot 8$ or 32, and that $2^5 = 32$.

Example 3 $\mathbf{2^4 \cdot 2^5 =} (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
$= 2^9$

Have students verify this result by showing that $2^4 \cdot 2^5 = 16 \cdot 32$ or 512, and that $2^9 = 512$.

In both examples, point out that the exponent in the result is the sum of the exponents of the two factors that were multiplied. In Example 2, we have $5 = 2 + 3$, and in Example 3, we have $9 = 4 + 5$.

Why Is This True?

Remind students that exponents are a "shorthand" method for indicating a repeated product of the same number or variable. So,

$$2^2 \cdot 2^3 = \underbrace{2 \cdot 2}_{\text{2 factors}} \cdot \underbrace{2 \cdot 2 \cdot 2}_{\text{3 factors}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 factors}} = 2^5$$

In general, when we multiply x^m and x^n, we will get

$$x^m \cdot x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{m \text{ factors}} \cdot \underbrace{x \cdot x \cdot \cdots \cdot x}_{n \text{ factors}} = \underbrace{x \cdot x \cdot \cdots \cdot x}_{m + n \text{ factors}} = x^{m+n}$$

Key Idea	When we multiply a power of x times another power of x, the result is a power of x whose exponent is the sum of the exponents of the two factors. In symbols, $x^m \cdot x^n = x^{m+n}$.

This rule holds when x is any number or variable, and m and n are any whole numbers.

- $x^5 \cdot x^7 = x^{12}$, since $5 + 7 = 12$
- $7^9 \cdot 7^6 = 7^{15}$, since $9 + 6 = 15$

We can now use this idea to multiply monomials.

Example 4 $\mathbf{3x^5 \cdot 4x^2} = (3 \cdot 4) \cdot (x^5 \cdot x^2)$ *Commutative Property*

$= 12x^7$ $\quad x^5 \cdot x^2 = x^{5+2}$

Notice that we collected the constant factors together and the variable factors together.

Example 5 $\mathbf{9y^8 \cdot (-y^7)} = [9 \cdot (-1)] \cdot (y^8 \cdot y^7)$ *Commutative Property*

$= -9y^{15}$ $\quad y^8 \cdot y^7 = y^{8+7}$

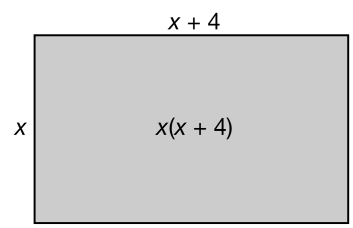
Note to the Teacher *Make sure the class receives plenty of opportunities to practice the multiplication of two monomials.*

Multiplying a Polynomial by a Monomial

Inform students that the Distributive Property is used to multiply a polynomial by a monomial. To illustrate the use of the Distributive Property when the quantities being multiplied involve the variable x, it will be helpful to do the following example on the chalkboard.

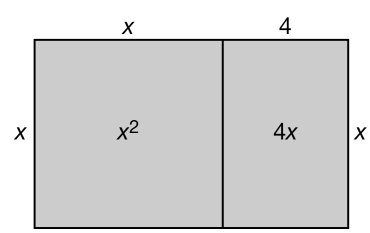
Example 6 **Compute the area of a rectangular region that has a width of x feet and a length of $(x + 4)$ feet.**

Solution Draw a rectangle and label the width and length as described.



Remind students that the area of a rectangle is found by multiplying its width and its length. So, the area of the rectangle is $x(x + 4)$.

Now draw the same rectangle again, with the length divided as shown below. Stress that the length of the rectangle is still $(x + 4)$ feet.



The area of the two regions that form this rectangle are $x \cdot x$ or x^2 and $4 \cdot x$ or $4x$, as shown above. So the area of the entire rectangle is $x^2 + 4x$.

Since the two figures are just different versions of the same rectangle, their areas are equal. Therefore, $x(x + 4) = x^2 + 4x$.

The area of the rectangle is $(x^2 + 4x)$ square feet.

This example illustrates the Distributive Property, which states that $a(b + c) = ab + ac$. The point to get across to your students is that this principle holds true even when a, b, and c are monomial expressions, meaning that $b + c$ is a polynomial.

Do several examples on the chalkboard. Here is a good one.

Example 7 **Find the product $2x^3(x^2 + 3x + 4)$.**

Solution Begin the multiplication by using the Distributive Property.

$$\begin{aligned} 2x^3(x^2 + 3x + 4) &= 2x^3(x^2) + 2x^3(3x) + 2x^3(4) \\ &= (2 \cdot 1)(x^3 \cdot x^2) + (2 \cdot 3)(x^3 \cdot x) + (2 \cdot 4)x^3 \\ &= 2(x^{3+2}) + 6(x^{3+1}) + 8x^3 \\ &= 2x^5 + 6x^4 + 8x^3 \end{aligned}$$