

Key Concepts

Lesson
6-3

Adding and Subtracting Fractions with Like Denominators

Objective Teach students to add and subtract fractions with like denominators.

Note to the Teacher *Doing arithmetic operations with fractions is perhaps the most difficult topic in mathematics learned by students up through grade 6. This concept is extremely important though, both from a conceptual point of view and as a basic skill. Fractions represent portions of a whole, and students will need to be able to manipulate fractions arithmetically. Here we will discuss arithmetic operations with fractions having the same denominators. It is very important to emphasize that the methods in this lesson apply only to fractions with like denominators.*

Adding Fractions with Like Denominators

Begin by presenting several examples of the types of problems that can be solved by adding fractions.

- If I have a portion of a pizza that is one third of the whole pizza and you have a portion that is one fifth of the pizza, what portion of the pizza do we have together?
- What part of a dollar do you have if you have a half dollar and a quarter in your pocket?
- How many total days is one third of a year together with one fourth of a year?

There are *algorithms* (procedures) for adding and subtracting fractions. In this lesson we will only be using the algorithm to add or subtract two fractions that have the same denominator. First let's visually interpret the sum of two fractions.

Example 1 What is the sum of $\frac{1}{5}$ and $\frac{2}{5}$?

Solution Help students visualize this problem by discussing it in terms of a pizza. Suppose a pizza has been cut into 5 equal pieces. Then each piece represents one fifth of the pizza, and

two fifths of the pizza is represented by any two pieces of the pizza. Suppose we take one of the pieces and place it on a plate. If we then place two of the remaining pieces on the plate with the first piece, we will have modeled the sum of $\frac{1}{5}$ and $\frac{2}{5}$. Since 3 of the 5 pieces of pizza are now on the plate, we have shown that

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}.$$

It may be helpful to students to use sketches to obtain a clearer understanding of the mathematics in Example 1. The fraction $\frac{1}{5}$ can be thought of as 1 of 5 blocks that are the same size, which together make up a whole.



Then the fraction $\frac{2}{5}$ can be thought of as 2 of 5 blocks just like those above.



Combined together, we have 3 blocks, which represents $\frac{3}{5}$ of the whole.



Example 2 Find $\frac{2}{7} + \frac{3}{7}$.

Solution Think of cutting a pizza into 7 equal pieces. Then $\frac{2}{7}$ of the pizza is represented by any 2 pieces, and $\frac{3}{7}$ of the pizza is represented by any 3 pieces. Taken together, these 5 pieces represent

$$\frac{2}{7} + \frac{3}{7} \text{ or } \frac{5}{7}$$

of the whole pizza.

Example 3 What is $\frac{3}{10} + \frac{7}{10}$?

Solution If a pizza is cut into 10 equal pieces and we take 3 pieces ($\frac{3}{10}$ of the pizza) together with 7 other pieces ($\frac{7}{10}$ of the pizza), we will have all 10 pieces of the pizza. So

$$\frac{3}{10} + \frac{7}{10} = \frac{10}{10} \text{ or } 1.$$

Note to the Teacher *After giving your students problems like those shown in Examples 1 and 2, ask your class if anyone sees a pattern concerning how fractions with like denominators are added. Guide the discussion to the conclusion that one may simply add the numerators of the two fractions and write the sum over the shared (common) denominator.*

Key Idea

When adding fractions with the same denominator, add the numerators and retain the denominator.

Example 4 Find $\frac{2}{9} + \frac{4}{9}$.

Solution

$$\begin{aligned} \frac{2}{9} + \frac{4}{9} &= \frac{2+4}{9} \\ &= \frac{6}{9} \end{aligned}$$

Why Does the Algorithm Work?

Suppose we want to add $\frac{a}{c} + \frac{b}{c}$. Have students again think in terms of a pizza, one that is cut into c equal pieces. So each piece is $\frac{1}{c}$ of the whole. We are adding a of these pieces to b of the pieces, for a total of $a + b$ pieces, each of which is $\frac{1}{c}$ of the whole. So we have $\frac{a+b}{c}$ parts of the whole. In symbols,

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}.$$

Subtract Fractions with Like Denominators

The algorithm for subtracting fractions with like denominators is similar to the algorithm for the addition of two fractions with like denominators.

Key Idea

When subtracting fractions with the same denominator, subtract the numerators and retain the denominator.

Example 5 Find $\frac{8}{11} - \frac{5}{11}$.

Solution $\frac{8}{11} - \frac{5}{11} = \frac{8-5}{11}$
 $= \frac{3}{11}$

Note to the Teacher *Be sure to emphasize that the algorithms given in this lesson only work when adding and subtracting fractions whose denominators are the same.*

