

Key Concepts

Lesson
7-4

Dividing Fractions

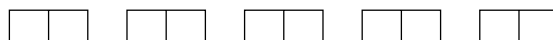
Objective Teach students the procedure for dividing one fraction by another.

Note to the Teacher *Dividing fractions is somewhat difficult conceptually. It is a good idea to first explain the process used to divide a whole number by a fraction whose numerator is 1, and then use that discussion to motivate the concept of reciprocal. This will help students see that dividing by a fraction is the same as multiplying by its reciprocal. We can then develop the method for dividing any pair of fractions.*

Make the following distinction between the division of whole numbers and the division of fractions. When one whole number is divided by another, there may be a remainder since one whole number cannot always be divided evenly into another whole number. However, there is never a remainder when two fractions are divided; the result is always another fraction or a whole number.

Dividing a Whole Number by a Fraction Whose Numerator is 1

Begin by recalling how we think about the division of whole numbers. One approach is to ask ourselves how many collections of size equal to the divisor are contained in a group whose size is equal to the dividend. For example, we know $6 \div 2 = 3$ because we know that a group of 6 items can be separated into 3 collections each containing 2 items. Now let's apply the same thought process to the division $5 \div \frac{1}{2}$. We can ask ourselves how many "collections" containing $\frac{1}{2}$ of an item are there in a group of 5 items. A model of this situation, showing five rectangles each divided into two equal parts, is shown below. (*Note:* The rectangles must be the same size.)



If each of the five larger rectangles represents 1 unit, then each of the smaller rectangles represents $\frac{1}{2}$ unit. So, the number of smaller rectangles is the number of "collections" containing $\frac{1}{2}$ of an item that

can be found in a group of 5 items. Since there are 10 smaller rectangles in the model, this shows that $5 \div \frac{1}{2} = 10$.

Example 1 What is $2 \div \frac{1}{4}$?

Solution Draw two rectangles, each divided into four equal parts.



If each larger rectangle represents 1 unit, then each smaller rectangle represents $\frac{1}{4}$ unit. Since there are 8 smaller rectangles in the model, this shows that

$$2 \div \frac{1}{4} = 8.$$

Note to the Teacher *At this point, consider having your students work in groups and provide each group with several problems like the one given in Example 1. Have the groups solve each problem by drawing a model. Then have a classroom discussion to see if any students see a pattern in the solutions to the problems. Lead the discussion to the observation that in each problem the answer can be obtained by multiplying the whole number by the denominator of the fraction.*

We know that a fraction indicates the division of the numerator by the denominator. For example, $\frac{7}{9} = 7 \div 9$ and conversely $7 \div 9 = \frac{7}{9}$. But we also know that $7 \times \frac{1}{9} = \frac{7}{9}$. Lead students to see that if $7 \div 9$ and $7 \times \frac{1}{9}$ are both equal to $\frac{7}{9}$, then they must also be equal to each other. That is, $7 \div 9 = 7 \times \frac{1}{9}$.

Multiply by the Reciprocal

Two numbers are *reciprocals* if their product is 1. Since $9 \times \frac{1}{9} = \frac{9}{9}$ or 1, this means that 9 and $\frac{1}{9}$ are reciprocals.

- The numbers 4 and $\frac{1}{4}$ are reciprocals, since $4 \times \frac{1}{4} = \frac{4}{4}$ or 1.
- The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals, since

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} \text{ or } 1.$$

Key Idea

The reciprocal of any fraction can be obtained by inverting the fraction. In other words, the denominator of the original fraction becomes the numerator and the numerator of the original fraction becomes the denominator. In symbols, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

- $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$.
- $\frac{2}{7}$ is the reciprocal of $\frac{7}{2}$.
- $\frac{8}{3}$ is the reciprocal of $\frac{3}{8}$.

Earlier it was stated that since $7 \div 9$ and $7 \times \frac{1}{9}$ are both equal to $\frac{7}{9}$, then $7 \div 9 = 7 \times \frac{1}{9}$. Also, we know that 9 and $\frac{1}{9}$ are reciprocals. So, dividing by a number and multiplying by the reciprocal of that number give the same result.

In Example 1, we found that $2 \div \frac{1}{4} = 8$. Since 2×4 is also 8, this means that $2 \div \frac{1}{4} = 2 \times 4$. So, dividing by a fraction and multiplying by its reciprocal give the same result.

Key Idea

Dividing a number by a fraction is equivalent to multiplying the number by the reciprocal of the fraction.

Note to the Teacher *Remind students of the class discussion on dividing whole numbers by fractions with numerator 1, when they found, for instance, that dividing by $\frac{1}{4}$ is the same as multiplying by 4. Since 4 is the reciprocal of $\frac{1}{4}$, that earlier discussion confirms this idea.*

Dividing Fractions

We have just seen that division by a fraction is the same as multiplication by the reciprocal of that fraction, and that the reciprocal is found by *inverting* the fraction. So, when dividing by a fraction, we need to invert the fraction and change the division symbol to multiplication. Then we complete the multiplication using the skills shown in Lesson 7-2.

Example 2 What is the value of $3 \div \frac{2}{7}$?

Solution Change from dividing by $\frac{2}{7}$ to multiplying by its reciprocal, $\frac{7}{2}$.
Then multiply.

$$\begin{aligned} 3 \div \frac{2}{7} &= 3 \times \frac{7}{2} \\ &= \frac{3 \times 7}{2} \\ &= \frac{21}{2} \text{ or } 10\frac{1}{2} \end{aligned}$$

Example 3 Find $2 \div \frac{9}{11}$.

Solution $2 \div \frac{9}{11} = 2 \times \frac{11}{9}$ *Change to multiplication by the reciprocal.*
 $= \frac{22}{9}$ or $2\frac{4}{9}$ *Write the improper fraction as a mixed number.*

The same procedure works when dividing one fraction by another fraction.

Key Idea

To divide one fraction (the dividend) by another (the divisor), multiply the dividend by the reciprocal of the divisor. In

symbols, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$.

Example 4 What is the value of $\frac{2}{3} \div \frac{3}{5}$?

Solution $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3}$ *Multiply by the reciprocal of the divisor.*
 $= \frac{2 \times 5}{3 \times 3}$ *Multiply numerators and denominators separately.*
 $= \frac{10}{9}$ or $1\frac{1}{9}$ *Write the improper fraction as a mixed number.*

Example 5 Find $\frac{3}{7} \div \frac{5}{6}$.

Solution $\frac{3}{7} \div \frac{5}{6} = \frac{3}{7} \times \frac{6}{5}$ *Multiply by the reciprocal of the divisor.*
 $= \frac{3 \times 6}{7 \times 5}$ *Multiply numerators and denominators separately.*
 $= \frac{18}{35}$

Exercises

Find each quotient. Write in simplest form.

1. $3 \div \frac{1}{7}$ **21**

2. $4 \div \frac{3}{8}$ **$10\frac{2}{3}$**

3. $\frac{1}{5} \div \frac{4}{9}$ **$\frac{9}{20}$**

4. $\frac{7}{10} \div \frac{4}{5}$ **$\frac{7}{8}$**

5. $\frac{2}{3} \div 8$ **$\frac{1}{12}$**

6. $\frac{5}{8} \div \frac{5}{12}$ **$1\frac{1}{2}$**

