Key Concepts

Lesson 8-2

Adding Integers

Objective Teach students to add both positive and negative integers.

Note to the Teacher *In the next four lessons of the Student Edition, your students will learn how to add, subtract, multiply, and divide integers. Conceptually, the most difficult idea for students to grasp is that of negative numbers and why there are special rules for doing arithmetic with them. So it is a good idea to begin this lesson by having a classroom discussion about negative numbers.*

Negative Numbers

Historically, the recognition and use of negative numbers developed very late. Despite the fact that mathematicians in ancient civilizations did perform subtraction, the recognition of negative numbers as “legitimate” numbers did not occur until the 1600s. Negative numbers are an extremely useful tool for many kinds of problems. For instance, the concept of a negative temperature, the notion of a negative balance in a bank account, and the use of negative numbers to describe the motion of an object in a particular direction are important uses of negative numbers.

The following examples show specific situations where negative numbers are used. Discuss these situations with your students, and then ask them if they know of other situations where negative numbers might be used.

- On the Celsius temperature scale, the freezing point of water is 0°, while on the Fahrenheit scale the freezing point of water is 32°. When we want to represent a temperature in Celsius degrees (written °C) that is colder (less than) the freezing point of water, we represent it by a negative number of degrees. For instance, a temperature that is 7 degrees *colder* than the freezing point of water on the Celsius scale would be represented using the negative number −7, and written as −7°C.
- When measuring changes in a quantity, such as a change in temperature or a change in rainfall from one year to the next, we use a negative number to describe a change in which the quantity decreases. For instance, if the temperature at noon is 20°C and the temperature at 6 P.M. is 12°C, we say that the change in temperature from noon to 6 P.M. is −8°C.

- When recording the assets of corporations, a deficit (a situation where the corporation owes more money than it has assets) can be described by saying that the company has negative assets. So, for instance, if the corporation has \$3,000,000 in assets but owes \$4,000,000, then we say the corporation has assets of “negative one million dollars,” written as −\$1,000,000.

After this discussion about how negative numbers are used, your students should be ready to learn how to add negative integers to positive integers. Begin by explaining the following basic rules.

Rules for Addition of Negative Integers

Remind students that the integers −4 and 4 are called **opposites** because they are the same distance from 0 in opposite directions on a number line.

Key Idea

Adding a negative integer to a positive integer gives the same result as subtracting the corresponding positive integer.

$$9 + (-4) = 9 - 4 = 5$$

opposites

In the Key Idea box above, the sum of 9 and −4 is shown to be the positive integer 5. Point out however that when we have an addition problem involving one positive integer and one negative integer, the sum might be a negative integer. For example,

$$5 + (-7) = 5 - 7 = -2 \quad \text{and} \quad 14 + (-21) = 14 - 21 = -7.$$

Key Idea

When we add two negative integers, the result is the opposite of what we would get if we added the corresponding positive integers.

$$(-2) + (-3) = -(2 + 3) = -5$$

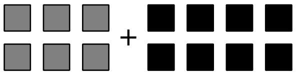
You should now explain why these rules are true. They are best explained using manipulatives like shaded squares, or colored counters as illustrated in the Student Edition.

Why Are These Rules for Addition True?

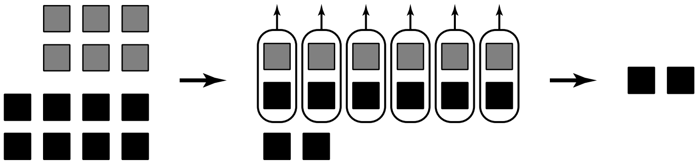
In the following discussions, positive integers are represented by collections of gray squares and negative integers are represented by collections of black squares. The figures below represent 6 and −8.

and

To model the addition of 6 and −8, we can combine the collections of gray and black squares. So, the model below represents 6 + (−8).

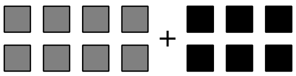


Remember that we are thinking of negative numbers as a deficit, so the collections above might be thought of as a collection of 6 “dollars” together with a deficit of 8 “dollars.” The 6 dollars will pay off part of the deficit of 8 “dollars,” but there will still be a deficit of 2 “dollars” remaining. This result can be shown using the squares if we cancel one black square and one gray square together by removing the pair.

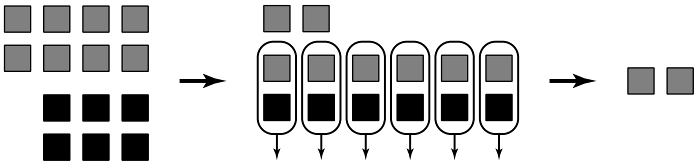


In the figure above, six pairs of gray and black squares were removed. When modeling the addition of integers, we remove pairs of black and gray squares until we have a collection that consists of squares of just one color.

Here is how we model the addition 8 + (−6).



Now we remove six pairs of squares of different colors.

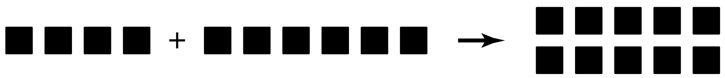


There are 2 gray squares remaining. This means that 8 + (−6) = 2.

The following example shows the sum of two negative integers. Your students should find this model easier to understand since all of the squares are the same color.

Example 1 **Add (−4) + (−6).**

Solution Model −4 as a collection of 4 black squares and −6 as a collection of 6 black squares. When we merge the two collections to model the addition, we get this model.



The result shown in the model is 10 black squares, so (−4) + (−6) = −10.

Adding Opposites

Key Idea	When we add a positive number to the opposite negative number, we always get zero.

The following example will make it clear to your students why this rule is true.

Example 2 **Use a model to demonstrate that 5 + (−5) = 0.**

Solution To see this, we represent 5 + (−5) as 5 gray squares together with 5 black squares. When the gray and black squares are removed in pairs, we are left with no squares at all. This means that the sum is 0.