

Key Concepts

Lesson
10-1

Ratios and Rates

Objective Teach students the concepts of ratios and rates, and how to use them in calculations.

Note to the Teacher *In this lesson, students will be introduced to ratios and rates. It is important that students learn the meanings of these concepts and then be given opportunities to use rates and ratios in computations. Besides solidifying their understanding of these two concepts, practice in calculating ratios and rates will reinforce their multiplication and division skills.*

Ratios

Begin by telling your students that a **ratio** is a way of comparing two numbers. Provide the following examples.

- The ratio of 1 to 2 is equivalent to the ratios 3 to 6 and 120 to 240.
- The ratio of 2 to 3 is equivalent to the ratios 8 to 12 and 160 to 240.
- The ratio of 5 to 3 is equivalent to the ratios 15 to 9 and 200 to 120.

Point out that the ratio of a to b can be expressed in several ways: a to b , $a:b$, a out of b , and $\frac{a}{b}$.

Now present the following example.

Example 1 All of the girls in Ms. Smith's class have either brown hair or blonde hair. There are 15 girls in the class and 5 of them have blonde hair. What is the ratio of blonde-haired girls to brown-haired girls?

Solution There are 5 blonde-haired girls, and 15 girls in all. So there are $15 - 5$ or 10 girls who are not blonde, and therefore have brown hair. The ratio of blonde-haired girls to brown-haired girls is $\frac{5}{10}$ or $\frac{1}{2}$. We may also write this ratio as 1:2.

Point out that a ratio is usually written as a fraction in simplest form. This is why the ratio of blonde-haired girls to brown-haired girls in Example 1 is given as $\frac{1}{2}$.

Note to the Teacher *It is important that students become comfortable calculating ratios. This will reinforce their skills with fractions. Many times it is useful to express a ratio as a fraction in simplest form. Here are some examples of those types of problems. Present one in class, and have your students do at least two more individually during class time. You might also want to give some more of these problems as homework. There are many of these types of problems in the Student Edition.*

Example 2 A baseball pitcher pitched in 25 games during the season, and the team won 15 of the games. Express the ratio of the games won out of the total number of games he pitched as a fraction in simplest form.

Solution The ratio of games won when he was pitching out of the total number of games he pitched is 15 to 25, which we can write as the fraction $\frac{15}{25}$. The greatest common factor (GCF) of the numerator and the denominator is 5. So,

$$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}.$$

The ratio in simplest form is $\frac{3}{5}$.

Example 3 Sally's mother bought two dozen pieces of fruit at the market. Six of the pieces of fruit were apples. Express the ratio of the number of apples to the total number of pieces of fruit as a fraction in simplest form.

Solution Since a dozen is 12 items, Sally's mother bought 2×12 or 24 pieces of fruit. The ratio of the number of apples out of the total number of pieces of fruit is then $\frac{6}{24}$. The GCF of the numerator and the denominator is 6, so the ratio in simplest form is $\frac{1}{4}$.

Example 4 In the school band there are 12 clarinet players out of 44 band members. Express the ratio of the number of clarinet players out of the total number of band members as a fraction in simplest form.

Solution The ratio of clarinet players to total band members is $\frac{12}{44}$. Since the GCF of the numerator and the denominator is 4, this ratio expressed in simplest form is $\frac{3}{11}$.

Rates

A **rate** is the ratio of two measurements that have different units. Rates are usually expressed in *per unit* form, which is a fraction whose denominator is 1.

Note to the Teacher *Dealing with measurements that have units can be confusing for some students. The following example involves units that should be familiar to your students.*

Example 5 Steve drove his car 90 miles to visit his grandparents. It took him 2 hours to drive this distance. Express the ratio of the miles he drove to the number of hours it took him as a rate in per unit form.

Solution The ratio is 90 miles to 2 hours, which can be written as the rate $\frac{90 \text{ miles}}{2 \text{ hours}}$. Since the per unit form of a rate has a denominator of 1, divide the numerator and denominator of this rate by 2.

$$\frac{90 \text{ miles}}{2 \text{ hours}} = \frac{(90 \div 2) \text{ miles}}{(2 \div 2) \text{ hours}} = \frac{45 \text{ miles}}{1 \text{ hour}}$$

So the rate at which Steve drove his car is 45 miles per hour.

Point out that the rate found in Example 5 is the average rate for the trip. Stress that the result does not mean Steve drove exactly 45 miles per hour during the entire 90-mile trip. The following example shows how rates can be used to make comparisons.

Example 6 A fast-food restaurant sells a 12-ounce soda for \$1.20 and a 16-ounce soda for \$1.40. Which of these sodas has a lower cost per ounce?

Solution To solve this problem, we need to find the cost per ounce for each size and then compare these rates. For the 12-ounce soda, the rate is

$$\frac{\$1.20}{12 \text{ ounces}} = \frac{\$0.10}{1 \text{ ounce}} \text{ or } 10 \text{ cents per ounce.}$$

For the 16-ounce soda, the rate is

$$\frac{\$1.40}{16 \text{ ounces}} = \frac{\$0.0875}{1 \text{ ounce}} \text{ or } 8.75 \text{ cents per ounce.}$$

Therefore, the 16-ounce soda is less expensive per ounce.

Lead a classroom discussion about the difference between ratios and rates. Guide the discussion so that students understand that a *ratio* is simply the quotient of two numbers, while a *rate* is the ratio of two measurements that have different units (like miles and hours, or dollars and ounces). Reinforce the fact that a rate is usually expressed in *per unit* form, where the denominator is 1.

Note to the Teacher *In order for your students to understand these new concepts well, they need to be given lots of practice problems that involve these concepts. Some of these problems may be done in small groups, but some should be done individually.*

