Key Concepts

Lesson 10-2

Solving Proportions

Objective Teach the concept of proportion, and solve proportion problems.

Note to the Teacher *In this lesson, students will be introduced to the concept of proportion. This topic is related to the concept of ratio, and studying proportions will help to solidify students' understanding of ratios. Students will also be asked to solve proportion problems using the technique of cross multiplication. This will help develop their skills with fractions. In this lesson, students will also see the use of several fundamental algebra techniques.*

Proportions

An equation that shows two ratios are equal is called a **proportion**. The equation $\frac{x}{y} = \frac{a}{b}$ is a proportion. Point out that neither y nor b can be zero. This proportion is sometimes also written as $x:y = a:b$ (read as "x is to y as a is to b."). Solving proportion problems usually involves the use of algebra, because one of the four values in the proportion is unknown and is represented by a variable. To solve these kinds of problems we use the following fact, which should be explained carefully to students.

Key Idea	Suppose $y \neq 0$ and $b \neq 0$. If $\frac{x}{y} = \frac{a}{b}$, then $xb = ya$.

In the Key Idea above, the values xb and ya are called the **cross products** of the proportion. The process of finding the cross products of a proportion is called **cross multiplication**.

Example 1 **Solve $\frac{3}{5} = \frac{a}{15}$.**

Solution **Step 1** Use cross multiplication.

$$\frac{3}{5} = \frac{a}{15} \quad \rightarrow \quad 3 \cdot 15 = 5 \cdot a$$

Step 2 Solve the equation from Step 1.

$3 \cdot 15 = 5 \cdot a$ *Write the cross products.*

$45 = 5a$ *Divide each side by 5.*

$9 = a$

Step 3 Verify the solution. When a is replaced by 9, the ratio on the right side of the given proportion becomes $\frac{9}{15}$. Dividing numerator and denominator by their GCF, 3, gives the other ratio, $\frac{3}{5}$.

Why Does Cross Multiplication Work?

Ask your students why they think that cross multiplication works. Point out that a justification can be derived by using their knowledge about subtraction of fractions.

Emphasize that the proportion $\frac{x}{y} = \frac{a}{b}$ is an equation. Show students that if the fraction $\frac{a}{b}$ is subtracted from both sides of the equation $\frac{x}{y} = \frac{a}{b}$, the result is $\frac{x}{y} - \frac{a}{b} = 0$. Point out that the left side of this equation involves the subtraction of two fractions with unlike denominators. Remind students that the first step when subtracting two fractions with unlike denominators is to rewrite the fractions so they have the same denominator. Multiplying $\frac{x}{y}$ by $\frac{b}{b}$ and multiplying $\frac{a}{b}$ by $\frac{y}{y}$ will result in a like denominator of *yb*.

$$\frac{x \cdot b}{y \cdot b} - \frac{y \cdot a}{y \cdot b} = 0$$

Now we need only subtract the numerators and retain the like denominator.

$$\frac{xb - ya}{yb} = 0$$

But a fraction equals zero only when its numerator equals zero. So, the value of the expression $xb - ya$ must equal zero. This only occurs when $xb = ya$. Therefore, if a proportion is true, its cross products must be equal.

Note to the Teacher *The above discussion about why the cross multiplication works is a very important example of mathematical reasoning. Namely it uses a computational technique (subtraction of fractions) to derive a general principle (the cross products of a proportion are equal). In order to solidify your students' understanding of these concepts, you should do several examples showing how this principle is applied. Do the following example in class, and then assign several other examples (including exercises in the Student Edition) to your students to work on individually or in small groups.*

Example 2 **In the New Hampshire presidential primary election of 2000, the ratio of Republicans to Democrats casting ballots was 5 : 4. It was reported that 200,000 Democrats voted. How many Republicans voted?**

Solution **Step 1** Let R be the number of Republicans who voted. From the given information, we can write a proportion. Be sure to write both ratios in the proportion in the form $\frac{\text{Republicans}}{\text{Democrats}}$.

$$\frac{5}{4} = \frac{R}{200{,}000}$$

Step 2 Solve for R.

$5 \cdot 200{,}000 = 4 \cdot R$ *Cross multiply.*

$1{,}000{,}000 = 4R$

$250{,}000 = R$ *Divide each side by 4.*

So, 250,000 Republicans voted in the primary.

Note to the Teacher *Ratio and proportion problems are fun for students because they deal with real-world situations. However, they tend to be difficult because of the basic algebra skills that are used in solving them. It will be beneficial if your students work together in small groups on word problems involving ratios and proportions. Ask each group to present their solutions to the rest of the class.*