

Key Concepts

Lesson
10-5

Percents and Fractions

Objective Teach students the concept of percent, how percents are used, and how they are related to fractions.

Note to the Teacher *In this lesson, your students will learn about the concept of percent and how to calculate a percent. Many of your students will have seen percents used. For example, they may know that 100% refers to an entire amount, 50% refers to half of the amount, and so on. Begin a classroom discussion using these examples, and see if your students can describe the meaning of percent. Lead the discussion to the following outcome.*

Definition of Percent

Like fractions, percents represent a part of a whole. In the case of a fraction like $\frac{3}{8}$, the denominator (in this case 8) represents the number of pieces into which the whole is divided, and the numerator (in this case 3) represents how many of the pieces we have. In other words, $\frac{3}{8}$ represents a collection of 3 out of 8 equal parts of a whole. Ask students to imagine a pizza cut into 8 equal slices. The fraction $\frac{3}{8}$ could then represent the part of the pizza eaten by a student who ate 3 slices of the pizza. A percent can be thought of in a similar way, except *the whole is always divided into 100 equal parts*. For example, 1 percent (written 1%) is $\frac{1}{100}$ of the whole. In other words, a **percent** is a ratio that compares a number to 100.

Writing Percents as Fractions

Point out that since a percent is a ratio, it can always be expressed as a fraction. For example, $50\% = \frac{1}{2}$. Explain that in order to express a percent as a fraction in simplest form, students should first express the percent as a fraction with denominator 100 and then simplify the fraction.

Example 1 Express 25% as a fraction in simplest form.

Solution 25% means 25 out of 100, and $\frac{25}{100} = \frac{1}{4}$.

Example 2 Express 20% as a fraction in simplest form.

Solution 20% means 20 out of 100, and $\frac{20}{100} = \frac{1}{5}$.

Example 3 Express 10% as a fraction in simplest form.

Solution 10% means 10 out of 100, and $\frac{10}{100} = \frac{1}{10}$.

Stress that we can have percents that are greater than 100%. These percents can also be written as fractions in simplest form. For example, 150% means $\frac{150}{100}$, which simplifies to $\frac{3}{2}$.

Note to the Teacher Give your students several percent values and have them convert the percents to fractions in simplest terms. First provide several easier examples like the ones in Examples 1–3 and then give a few more difficult examples like those in Examples 4 and 5 below.

Example 4 Express 32% as a fraction in simplest form.

Solution 32% means 32 out of 100 or $\frac{32}{100}$. To simplify this fraction, we find the greatest common factor (GCF) of the numerator 32 and the denominator 100. The GCF of 32 and 100 is 4. Dividing the numerator and denominator by the GCF,

$$\frac{32}{100} = \frac{32 \div 4}{100 \div 4} = \frac{8}{25}$$

So, 32% expressed as a fraction in simplest form is $\frac{8}{25}$.

Example 5 Express 85% as a fraction in simplest form.

Solution 85% means 85 out of 100 or $\frac{85}{100}$. The GCF of 85 and 100 is 5. Dividing by the GCF gives

$$\frac{85}{100} = \frac{85 \div 5}{100 \div 5} = \frac{17}{20}$$

So, 85% expressed as a fraction in simplest form is $\frac{17}{20}$.

Writing Fractions as Percents

Now explain to your students that the process shown in Examples 1–6 is reversible. In other words, not only can we express a percent as a fraction, but we can express a fraction as a percent. To do this, remind your students that percent means a ratio that compares a number to 100. So to convert a fraction to a percent, we set up a proportion problem that compares the given fraction to a fraction whose denominator is 100 but whose numerator is unknown.

Example 6 is an example that your students will already know without seeing the calculations, but this makes it a good example for focusing students' attention on the calculations themselves.

Example 6 Express the fraction $\frac{1}{4}$ as a percent.

Solution Step 1 Set up a proportion. Let x represent the unknown numerator (the percent).

$$\frac{1}{4} = \frac{x}{100}$$

Step 2 Solve the proportion using cross multiplication.

$$1 \cdot 100 = 4 \cdot x \quad \text{Write the cross products.}$$

$$100 = 4x \quad \text{Divide each side by 4.}$$

$$25 = x$$

So, $\frac{1}{4}$ is equivalent to 25%.

Example 7 Express the fraction $\frac{3}{5}$ as a percent.

Solution Set up a proportion and solve it.

$$\frac{3}{5} = \frac{x}{100}$$

$$3 \cdot 100 = 5 \cdot x$$

$$300 = 5x$$

$$60 = x$$

So, $\frac{3}{5}$ is equivalent to 60%.

Give your students several problems like Examples 6 and 7 to work on by themselves or in small groups. It is a good idea to use only fractions whose denominators are factors of 100, such as 2, 4, 5, 20, and 25. Then the resulting equivalent percent will be a whole number.

Here is an application problem that you can have your students do in class.

Example 8 In a class of 20 students, there are 13 girls. What percent of the class is girls?

Solution The percent of girls in the class is the ratio of girls to total students. So we can write the proportion

$$\frac{13}{20} = \frac{g}{100},$$

where g represents the percent of girls in the class. Solve this proportion for g .

$$13 \cdot 100 = 20 \cdot g \quad \text{Cross multiply.}$$

$$1,300 = 20g \quad \text{Divide each side by 20.}$$

$$65 = g$$

So, 65% of the students in the class are girls.

Note to the Teacher *Computing with percents is an extremely important skill. Be sure to provide your students with numerous problems to work. This will help them solidify their understanding of the concept of percent, as well as help with their computational skills involving fractions.*

