# Key Concepts



## Angles

**Objective** Teach students how to measure and classify angles, and the relationships between pairs of angles.

**Note to the Teacher** *This lesson begins the discussion of plane geometry, the study of two-dimensional geometric figures. Plane geometry is one of the oldest areas of mathematics. The treatment of plane geometry we teach our students today has its roots in the work of Thales, Pythagoras, Euclid, and Archimedes in ancient Greece, between 2,200 and 2,600 years ago.*

The basic figure studied in this lesson is an *angle*, but even more fundamental figures are points and line segments. Your students will most likely have intuitive ideas about what points, line segments, and angles are. You might want to have a classroom discussion about their mathematical meanings. Here are the basic ideas to guide that discussion.

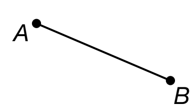
### Points, Line Segments, and Angles

Three of the basic figures in elementary plane geometry are points, line segments, and angles.

A **point** is an exact location in the plane. Emphasize that a point has neither shape nor size. We usually denote points by capital letters and represent them with dots.

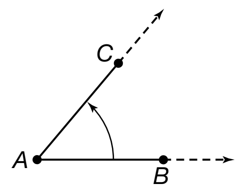

Stress to students that even though dots made on paper or on the chalkboard do have a size and shape to them, a dot is the best physical representation we can use for a point.

Given two points *A* and *B*, the **line segment *AB*** (written in symbols as $\overline{AB}$) refers to the part of a straight line that connects the points *A* and *B* and includes points *A* and *B*.




Points *A* and *B* are called the **endpoints** of $\overline{AB}$.

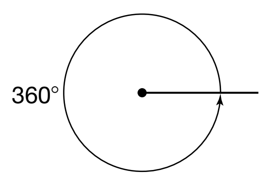
A more difficult concept for students to grasp is that of an **angle**. Two line segments that have a common endpoint, say $\overline{AB}$ and $\overline{AC}$, can be thought of as forming an angle. The common endpoint is called the **vertex** of the angle. Point out that the sides of an angle actually extend infinitely away from the vertex and do not end. The measure of an angle is the measure of how much one needs to rotate one of its sides in order to coincide with the other side. This rotation is measured in units called **degrees**.



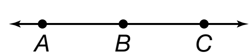

We can refer to the angle above as $\angle A$, as $\angle CAB$, or as $\angle BAC$. Point out that the vertex of the angle, point $A$ in the figure above, is the middle letter when using three letters to name an angle.

## Classifying Angles

There are 360 degrees (written 360°) in a full circular rotation.



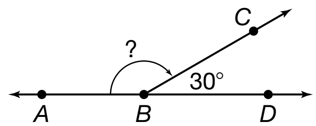

If points $A$, $B$, and $C$ all lie on a line, as in the figure below,




then $\angle ABC$ is said to be a **straight angle**, and its measure is one-half the measure of a full circular rotation or 180°. In symbols, this is written as $m\angle ABC = 180°$ (read "the measure of angle $ABC$ equals 180 degrees").

The following example will help your students understand how angle measures can be added and subtracted.

**Example** **Consider the figure below, where $m\angle CBD = 30°$. If $\angle ABD$ is a straight angle, what is the measure of $\angle ABC$?**




**Solution** The two angles, $\angle ABC$ and $\angle CBD$, together make up $\angle ABD$, which has a measure of 180° since the points *A*, *B*, and *D* all lie on a straight line. Written as an equation, we have

$$m\angle ABC + m\angle CBD = 180°.$$

But we know that $m\angle CBD = 30°$. So,

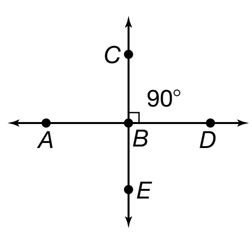
$$m\angle ABC + 30° = 180°$$

and therefore

$$m\angle ABC = 150°.$$

In the Example, $\angle ABC$ and $\angle CBD$ together made up the 180° angle, which is the angle measure of a straight angle. When two angles (like $\angle ABC$ and $\angle CBD$) have measures whose sum is 180°, they are called **supplementary angles**.

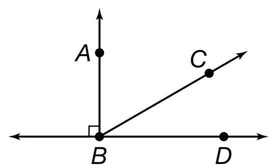
There are other types of angles that have special names. An angle whose measure is 90° is called a **right angle**. Point out to students that a right angle can be seen where a straight wall meets the floor. Ask students to point out some other places in the classroom where a right angle is formed. Since $90° + 90° = 180°$, any pair of right angles is a pair of supplementary angles. Conversely, if two angles form a straight angle and one of the angles measures 90°, then the other angle must also measure 90°. Visually, this means that if two lines intersect at a 90° angle as shown below,



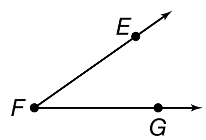

then $\angle ABC$, $\angle ABE$, and $\angle EBD$ also measure 90°. Point out that the symbol shown at the vertex of $\angle CBD$ indicates that the angle is a right angle. When two lines intersect at a right angle as in the figure above, the lines are said to be **perpendicular**.

Here are some other terms about angles for your students to learn.

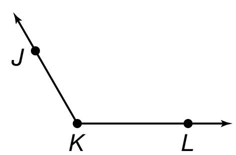
- Two angles are **complementary** if the sum of their measures is 90°.
- An angle is **acute** if its measure is greater than 0° but less than 90°.
- An angle is **obtuse** if its measure is greater than 90° but less than 180°.




$\angle ABC$ and $\angle CBD$ are complementary.




$\angle EFG$ is acute.




$\angle JKL$ is obtuse.

**Note to the Teacher** *Have your students use a protractor to measure several angles. You can use angles that are in the Student Edition or have students draw the angles themselves. Then have the students classify the angles as acute, right, or obtuse.*