Key Concepts

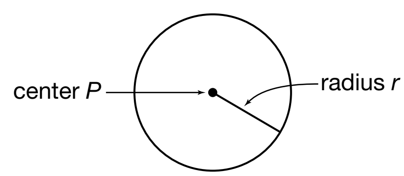
Area of Circles

Objective Teach students the formula for the area of a circle in terms of the number π, and how to compute the area of a circle using an approximation for π.

Note to the Teacher *In this lesson your students will learn the formula for the area of a circle of radius r, $A = \pi r^2$. They will also compute some areas, using an approximation for π. Since the number π is unfamiliar to them, its use may be confusing. One way to approach this lesson is to begin by leading a general classroom discussion about circles, and then talk about the number π and how to use it.*

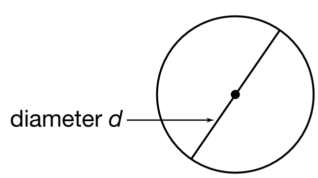
Circles and the Number π

The defining characteristics of a circle are its **center** and its **radius**. Namely, given a point P, then the circle with center P and radius r consists of the set of all points in the plane whose distance from point P is r units.



A line segment connecting the center point of a circle to any point on the circle is a radius. The length of such a segment is also referred to as the *radius*.

A line segment that goes through the center point and has its endpoints on the circle is called a **diameter** of the circle.



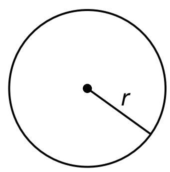
The term *diameter* is also often used to denote the length of this line segment. We will use the term for both meanings when the context is clear. Notice that the length of a diameter is equal to twice the length of a radius. If r is the length of a radius and d is the length of a diameter, then $d = 2r$.

The **circumference** of a circle is the distance around the circle. That is, if one imagines putting a string around the circle, then the length of the string is the circumference.

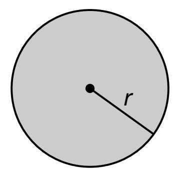
One of the great discoveries of the ancient Greeks is that there is a number, they called it π (the Greek letter for the "*p*" sound), such that the circumference of any circle is π times the diameter of the circle.

The circumference C of a circle of diameter d is given by the formula $C = \pi d$. Since the diameter d of a circle is twice the radius r, the circumference is also given by the formula $C = 2\pi r$.

The ancient Greeks also found that the *area* A of a circle of radius r (or more precisely, the area of the region lying within the circle) is given by the formula $A = \pi r^2$.



Circumference = πd or $2\pi r$

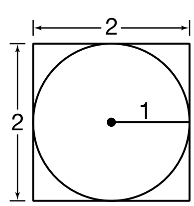


Area = πr^2

An Approximation for the Number π

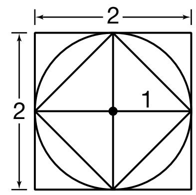
Another significant accomplishment of the ancient Greeks was their analysis of the value of the number π. From the area formula, we see that the area of a circle of radius 1 is π square units. Have students do the following activity, which will show that this area is somewhere between 2 and 4 square units.

Draw the figure shown below on the chalkboard. The figure shows a circle of radius 1 (and so its area is π) inside a square with side length 2.



The area of the square is 2×2 or 4. Since the entire circle is inside the square but parts of the square are not inside the circle, the area of the circle is less than the area of the square. Therefore, we can conclude that $\pi < 4$.

Now draw a square inside the circle so that the vertices of the small square are the points where the circle touches the large square. Draw line segments from the center of the circle to the four corners of the small square. We can see that the small square consists of four right triangles that are all the same size (so they all have the same area). But from the figure we can also see that the large square consists of eight of these right triangles. Point out that this means the area of the smaller square is exactly half the area of the large square. Since we know the area of the large square is 4, we can conclude that the area of the small square is 2. But the area of this small square is *less than* the area of the circle, so we can conclude that $2 < \pi$. Therefore, we have $2 < \pi < 4$.



The ancient Greeks carefully calculated approximations for the value of π. In particular, one good approximation is π is approximately equal to 3.1416. Another common approximation is π is approximately equal to $\frac{22}{7}$.

The Number π is Irrational

The ancient Greeks also discovered that π is an irrational number. Remind students that a number is rational if it can be written as a fraction (strictly speaking, in the form $\frac{p}{q}$, where p and q are integers). For example, $\frac{1}{2}$, $\frac{3}{5}$, $-\frac{2}{7} = \frac{-2}{7}$, $4 = \frac{4}{1}$, and $-2 = \frac{-2}{1}$ are all rational numbers. A number written using decimal notation like 0.201 is also a rational number since $0.201 = \frac{201}{1{,}000}$.

A number that is *not* a rational number is called an **irrational number**. Point out that besides π being irrational, other examples of irrational numbers include $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$. To actually *prove* that certain numbers are irrational is rather difficult, but the ancient Greeks were the first to show they exist, and in particular that π is irrational.

Area of a Circle

Stress that π is not *equal* to 3.1416, nor is it *equal* to $\frac{22}{7}$. Emphasize that these are only *approximations* of the value of π. This means, for example, that the *exact* answer to the question of what is the circumference of a circle of radius 3, is $2 \cdot \pi \cdot 3 = 6\pi$. It is only *approximately* equal to $6 \cdot 3.1416 = 18.8496$. Discussing these points in class will help students understand the role of estimation, or approximation, in addressing geometry problems involving circles. End the discussion by calculating the approximate area of a circle. Use the approximation 3.14 for π.

Example **Approximate the area of a circle of radius 14. Use 3.14 for π.**

Solution The formula for the area of a circle is $A = \pi r^2$.

$A \approx 3.14 \cdot 14^2$ *Substitute 3.14 for π and 14 for r.*

$\approx 3.14 \cdot 196$

≈ 615.44

The area of the circle is approximately 615.44.