

# Key Concepts

Lesson  
4-5

## Inequalities

**Objective** Teach students to solve and graph simple inequalities.

**Note to the Teacher** *It is important to emphasize that solving inequalities does not produce a single solution, but rather a range of solutions called the solution set. Emphasize that this is an important difference between inequalities and equations. Also stress the similarities between solving equations and solving inequalities, such as how we can add or subtract the same number from both sides of an inequality just as we do with equations.*

## Inequalities

An **inequality** is a mathematical sentence involving a variable and numbers that compares one expression to another using an inequality symbol. There are four inequality symbols:  $<$  (less than),  $>$  (greater than),  $\leq$  (less than or equal to), and  $\geq$  (greater than or equal to). Four examples of inequalities are shown below.

$$x \geq 3 \quad 2x + 5 < 3x + 3 \quad x + 5 \leq x \quad 5 - x > 3$$

We say a number *satisfies* an inequality if a true statement results when the number is substituted for the variable in the inequality.

**Example 1** Does the number 3 satisfy  $3 - x \leq 2$ ?

**Solution** Substitute 3 for the variable  $x$  in the inequality.

$$3 - x \leq 2$$

$$3 - 3 \leq 2 \quad \text{Replace } x \text{ with } 3.$$

$$0 \leq 2 \quad \text{Yes, } 0 \text{ is less than or equal to } 2.$$

The number 3 does satisfy the inequality  $3 - x \leq 2$ .

**Example 2** Does the number 2 satisfy  $1 + x > 5$ ?

**Solution** Substitute 2 for the variable  $x$  in the inequality.

$$1 + x > 5$$

$$1 + 2 > 5 \quad \text{Replace } x \text{ with } 2.$$

$$3 > 5 \quad \text{No, } 3 \text{ is NOT greater than } 5.$$

The number 2 does *not* satisfy the inequality  $1 + x > 5$ .

## Graphing the Solution Set of an Inequality

When we speak of *solving* an inequality, we mean finding its solution set. Emphasize the fact that solving an inequality means *describing a set of numbers*, not just finding a single number. The **solution set** of an inequality is the set of all values of the variable that satisfy the inequality.

To visualize the solution set of an inequality, we will graph it on a number line. To graph a solution set, we draw a number line and then shade the part of the number line corresponding to the solution set. Stress that all of the numbers (fractions, decimals, and so on) and not just the whole numbers on the shaded portion of the number line are included in the solution set. Point out that the solution set for each inequality in this lesson will be a “half-line,” which may or may not include the endpoint. When the endpoint is part of the solution set, we draw a dot for the endpoint, and when it is not part of the solution set, we draw a circle.

The figure below shows the graph of the solution set  $x \leq 5$ . The solution set includes all points to the *left* of 5 (less than 5) on the number line as well as the point at 5 (equal to 5). Stress that a dot is used for inequalities involving the symbols  $\leq$  and  $\geq$ .



The next figure shows the graph of the solution set  $x > 2$ . The solution set includes all points to the *right* of 2 (greater than 2) on the number line but *not* the point at 2. Stress that a circle is used for inequalities involving the symbols  $>$  and  $<$ .



## Solving Inequalities

In Lessons 4-2 and 4-3, students learned to solve one-step equations using the addition, subtraction, and division properties of equality. Then in Lesson 4-4, they learned to solve two-step equations using these same properties. Emphasize that solving inequalities and solving equations are similar processes. The best way to introduce solving inequalities is by doing several on the chalkboard, and then having students solve some additional ones.

Begin by formally introducing your students to the addition, subtraction, and division properties of inequalities. Point out the difference between the Division Property of Equality and the Division Property of Inequality.

<b>Addition Property of Inequality</b>	<p>Adding the same number to each side of an inequality produces an equivalent inequality. That is, the solution set of the resulting inequality is the same as the solution set of the original.</p> <p>For example,</p> $\begin{aligned} x - 2 &\leq 1 \\ x - 2 + 2 &\leq 1 + 2 \\ x &\leq 3 \end{aligned}$ <p>The inequality <math>x \leq 3</math> is equivalent to the original inequality <math>x - 2 \leq 1</math>.</p>
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<b>Subtraction Property of Inequality</b>	<p>Subtracting the same number from each side of an inequality produces an equivalent inequality.</p> <p>For example,</p> $\begin{aligned} x + 5 &> 12 \\ x + 5 - 5 &> 12 - 5 \\ x &> 7 \end{aligned}$ <p>The inequality <math>x &gt; 7</math> is equivalent to the original inequality <math>x + 5 &gt; 12</math>.</p>
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<b>Division Property of Inequality</b>	<p>Dividing each side of an inequality by the same <i>positive</i> number produces an equivalent inequality.</p> <p>For example,</p> $\begin{aligned} 3x &\geq 9 \\ \frac{3x}{3} &\geq \frac{9}{3} \\ x &\geq 3 \end{aligned}$ <p>The inequality <math>x \geq 3</math> is equivalent to the original inequality <math>3x \geq 9</math>.</p>
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**Note to the Teacher** *Point out to students that solving an inequality amounts to finding an equivalent inequality that has the variable with a coefficient of 1 alone on one side. This form of an equivalent inequality is easily graphed, showing the solution set of the original inequality.*

**Example 3** Solve  $x + 5 \leq 8$  and graph the solution.

**Solution** This is a one-step inequality. Subtract 5 from each side to obtain the solution.

$$\begin{aligned} x + 5 &\leq 8 \\ x + 5 - 5 &\leq 8 - 5 \\ x &\leq 3 \end{aligned}$$

So the solution is  $x \leq 3$ . The graph of this solution is all points to the left of 3 and also 3 itself. A dot is used at 3.



**Example 4** Solve  $x - 3 > 1$  and graph the solution.

**Solution**       $x - 3 > 1$

$x - 3 + 3 > 1 + 3$       *Add 3 to each side of the inequality.*

$x > 4$

So the solution is  $x > 4$ . The graph of this solution is all points to the right of 4 but *not* including 4. A circle is used at 4.

