

Key Concepts

Lesson
4-6

Functions and Graphs

Objective Informally introduce students to the concept of a function as a relationship between two variable quantities, and show them how to display the relationship as a coordinate graph.

Note to the Teacher *The idea of a function is introduced informally in this lesson. Of course, there is a precise mathematical definition, but it will not be presented at this time.*

Examples of Functions

Point out to students that there are many relationships between two variable quantities that we use on a regular basis in our daily lives. Three examples are given below.

- Suppose Joshua is driving his car from San Francisco, California, to Los Angeles, California, at a constant speed of 50 miles per hour and that he left San Francisco at 12 noon. There is a relationship between the Joshua's distance d in miles from San Francisco and the time t in hours since he began driving at 12 noon. The distance d is a function of the time t , because the more time Joshua spends driving, the further he is from San Francisco.
- When measuring temperature, we can use the Celsius scale or the Fahrenheit scale. If we know the Celsius temperature C of a liquid and want to convert this temperature to the Fahrenheit scale, we can use the formula $F = \frac{9}{5}C + 32$. In this formula, the Fahrenheit temperature is a function of the Celsius temperature. Whenever the variable C in the formula is replaced by an actual Celsius temperature, the value of F (the corresponding Fahrenheit temperature) can be found.
- Suppose h represents the average height of all girls of a certain age in the United States. The value of h therefore depends on the age, and so we say that the average height of a girl in the United States is a *function* of her age.

When one quantity depends on another, we call the quantity that depends on the other one the **dependent quantity** or the **dependent variable**. The quantity that the dependent variable depends on is called the **independent quantity** or the **independent variable**. We say the dependent variable is a *function* of the independent variable.

Refer students to the driving and temperature examples on the previous page. In the driving example, point out that the driving time is the independent quantity and the distance traveled from San Francisco is the dependent quantity. So we say *the distance from San Francisco is a function of time*. In the temperature formula, stress that the Fahrenheit temperature is the dependent quantity and the Celsius temperature is the independent quantity. We say *the Fahrenheit temperature is a function of the Celsius temperature*.

Note to the Teacher *The idea of the function actually being a relationship rather than a number is one that may be too abstract for some students. It is probably a good idea to mention this fact, but it does not need to be emphasized at this stage.*

Graphing Functions

When one quantity depends on another, it is often useful to show the relationship graphically. In order to graph the relationship between two variable quantities, we begin by writing a list of values for the independent quantity. Then we compute the value of the dependent quantity that corresponds to each value of the independent quantity we listed. We use these values to write ordered pairs of the form (independent quantity, dependent quantity). These ordered pairs can then be plotted on a coordinate plane, usually with the independent quantity represented along the horizontal axis and the dependent quantity represented along the vertical axis.

Example Graph the Celsius/Fahrenheit temperature relationship represented by the formula $F = \frac{9}{5}C + 32$.

Solution Step 1 Choose several values for the independent variable C . Here we will use 0, 25, 50, 75, and 100.

Step 2 Find the value of the dependent variable F corresponding to each chosen value for the independent variable.

Replace C with 0. $F = \frac{9}{5}(0) + 32$,
so $F = 0 + 32$, or 32

Replace C with 25. $F = \frac{9}{5}(25) + 32$,
so $F = 45 + 32$, or 77

Replace C with 50. $F = \frac{9}{5}(50) + 32$,
so $F = 90 + 32$, or 122

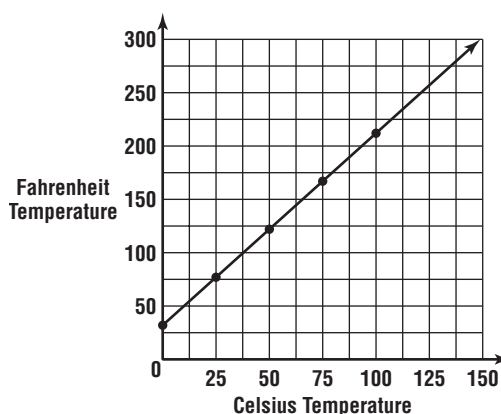
Replace C with 75. $F = \frac{9}{5}(75) + 32$,
so $F = 135 + 32$, or 167

Replace C with 100. $F = \frac{9}{5}(100) + 32$,
so $F = 180 + 32$, or 212

Step 3 Use the values from Steps 1 and 2 to write ordered pairs of the form (C, F) .

$(0, 32)$, $(25, 77)$, $(50, 122)$, $(75, 167)$, $(100, 212)$

Step 4 Plot the ordered pairs from Step 3 on a coordinate plane. Draw a line through the plotted points.



Depending on the context of the situation, a line or curve may be drawn through the plotted points on a coordinate plane to give us an idea about what happens between points which we have plotted. In the temperature situation discussed in Example 1, point out that there are obviously Celsius temperatures other than the five we chose to use. Each of these other Celsius temperatures has a unique Fahrenheit temperature associated with it, and these other pairs of temperatures are represented on the line drawn through the five plotted points in the graph.

