

Key Concepts

Lesson
5-3

Simplifying Fractions and Ratios

Objective Review the notion of equivalent fractions, and teach students how to find the simplest form of a fraction.

Note to the Teacher *Although this lesson teaches only a procedure for finding the simplest form of a fraction, it is important to review the idea of equivalent fractions, which is the underlying concept.*

Equivalent Fractions

Point out that sometimes different fractions (with different numerators and denominators) represent the same number. Fractions that name the same number are called **equivalent fractions**. For example, the fraction $\frac{4}{2}$ equals 2, since when we divide 4 units into two equal parts, each part contains 2 units. In the same way, the fraction $\frac{6}{3}$ also equals 2. The fractions $\frac{4}{2}$ and $\frac{6}{3}$ are *equivalent fractions*.

Example 1 Are $\frac{2}{3}$ and $\frac{4}{6}$ equivalent?

Solution We can model the fractions, using a separate rectangle for each one. It is very important that the two rectangles be the same size.



Notice that the same portion of the whole rectangle is shaded in both cases. This indicates that the fractions are equivalent. Another way to view this is to recognize that each of the parts (each third) of the rectangle on the left corresponds to exactly two parts (which are sixths) of the rectangle on the right. So the two shaded thirds of the left rectangle are exactly the same as the four shaded sixths of the right rectangle.

Point out to students that we do not always want to draw rectangles to compare fractions, because the denominators in the fractions might be quite large. Suggest that we need a procedure for comparing fractions that does not require the use of a model.

Key Idea

If one of two fractions can be obtained by multiplying both the numerator and the denominator of the other by the same whole number, then the two fractions are *equivalent*.

Since $\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$, the fractions $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent.

Why Is This True?

Refocus students' attention on Example 1, where the fractions $\frac{2}{3}$ and $\frac{4}{6}$ were modeled. Point out that the total number of smaller rectangles in each model is the denominator in the fraction it represents, and that the number of shaded rectangles is the numerator of the fraction. Emphasize that the model for $\frac{4}{6}$ can be formed by copying the model for $\frac{2}{3}$ and splitting each of the three smaller rectangles in half. Stress that the act of splitting each rectangle in the model for $\frac{2}{3}$ in half is representative of multiplying both the numerator (the number of shaded smaller rectangles) and the denominator (the total number of smaller rectangles) by 2.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Here are some examples.

- $\frac{5}{7}$ and $\frac{5 \times 3}{7 \times 3} = \frac{15}{21}$ represent the same number.
- $\frac{4}{19}$ and $\frac{4 \times 6}{19 \times 6} = \frac{24}{114}$ represent the same number.
- $\frac{6}{7}$ and $\frac{6 \times 10}{7 \times 10} = \frac{60}{70}$ represent the same number.

What these examples show is that if we multiply both the numerator and the denominator of a fraction by the same number, we obtain an equivalent fraction. In the same way, suppose for a given fraction, there is a whole number that divides both the numerator and the denominator evenly. Then the fraction that results is equivalent to the original fraction.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Here are some examples.

- The number 8 divides both the numerator and the denominator of the fraction $\frac{8}{24}$ evenly. When we divide 8 by 8, we get 1; when we divide 24 by 8, we get 3. This means that $\frac{8}{24}$ and $\frac{1}{3}$ are equivalent fractions.
- $\frac{10}{25}$ and $\frac{10 \div 5}{25 \div 5} = \frac{2}{5}$ are equivalent fractions.
- $\frac{14}{49}$ and $\frac{14 \div 7}{49 \div 7} = \frac{2}{7}$ are equivalent fractions.

Simplest Form of a Fraction

We say that a fraction is in **simplest form** when its numerator and denominator have no common factors other than 1. For example, the fractions $\frac{3}{5}$, $\frac{4}{7}$, $\frac{2}{9}$, and $\frac{1}{11}$ are all in simplest form, but the fractions $\frac{2}{4}$, $\frac{3}{9}$, $\frac{16}{24}$, and $\frac{30}{36}$ are *not* in simplest form.

Tell students that to simplify a fraction:

- they should divide its numerator and its denominator by any whole number that is a factor of both numbers;
- they should then look at the numerator and denominator of the resulting equivalent fraction to see if there is another whole number that evenly divides both numerator and denominator; and
- if so, they should do the division and examination steps again and again until the only number that divides both numerator and denominator is 1. When the only common factor is 1, the fraction is in simplest form.

Example 2 Express $\frac{30}{75}$ in simplest form.

Solution The number 5 is a common factor of the numerator and denominator. So, an equivalent fraction is $\frac{30 \div 5}{75 \div 5} = \frac{6}{15}$. But 3 is a common factor of 6 and 15, so another equivalent fraction is $\frac{6 \div 3}{15 \div 3} = \frac{2}{5}$. Since the only common factor of 2 and 5 is 1, the fraction $\frac{2}{5}$ is in simplest form.

Example 3 Express $\frac{28}{42}$ in simplest form.

Solution $\frac{28}{42} = \frac{28 \div 7}{42 \div 7} = \frac{4}{6}$ 7 is a common factor of 28 and 42.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3} \quad 2 \text{ is a common factor of 4 and 6.}$$

The fraction $\frac{28}{42}$ expressed in simplest form is $\frac{2}{3}$.

Stress that there is just one simplest form for any given fraction. This means that if two fractions have the same simplest form, then they are equivalent.

There is a systematic procedure for finding the simplest form of a fraction. To express a fraction in simplest form:

- find the greatest common factor (GCF) of the numerator and the denominator,
- divide the numerator and denominator by the GCF, and then
- write the resulting fraction.

Example 4 Find the simplest form of $\frac{21}{56}$.

Solution Step 1 Find the GCF of 21 and 56.

The factors of 21 are 1, 3, 7, and 21. The factors of 56 are 1, 2, 4, 7, 8, 14, 28, and 56. So, the GCF of 21 and 56 is 7.

Step 2 Divide the numerator and denominator by the GCF.

$$\frac{21}{56} = \frac{21 \div 7}{56 \div 7} = \frac{3}{8}$$

Step 3 Write the resulting fraction.

The simplest form of the fraction $\frac{21}{56}$ is $\frac{3}{8}$.

Note to the Teacher *The same procedure shown in this lesson works when dealing with ratios. Give students some problems that are stated in terms of ratios as well as in terms of fractions.*

