# Key Concepts

Lesson 5-4

## Fractions and Decimals

**Objective** Teach students to write fractions as decimals, using long division.

**Note to the Teacher** *This lesson is an excellent application of long division and division of decimals. Emphasize to students that it is important to be able to write fractions as decimals. Point out that it is more difficult to compare and perform addition and subtraction on fractions than on decimals.*

### Fractions Represent Division

To introduce the lesson, remind students that fractions really represent the division of one integer by another integer. For example, $\frac{5}{7}$ represents the division of 5 by 7. The following example should help students visualize this concept.

**Example 1** **A group of 7 friends decide to have pizza after seeing a movie. They agree to pool their money to pay for the pizzas. Altogether, they have enough money to buy 5 medium pizzas. Collin suggests they have each pizza cut into 7 equal pieces so that it will be easy to make sure everyone receives the same amount of pizza. Will Collin's idea work, and if so, what part of a whole pizza will each of the friends receive?**

**Solution** Using Collin's suggestion, each of the 5 pizzas will be cut into 7 equal pieces. There will be a total of $5 \times 7$ or 35 pieces, all of which are the same size. Then each of the 7 friends will receive $35 \div 7$, or 5 pieces, with no extra pieces left over. So, Collin's idea for dividing the pizzas equally will work. Since the pieces are all the same size, each of the 7 friends will receive $\frac{5}{7}$ of a whole pizza.

### Writing Decimals as Fractions

The division represented by a fraction can be performed to produce the decimal equivalent of the fraction.

| **Key Idea** | To write a fraction in decimal form, divide the numerator by the denominator. |
|---|---|

**Example 2** **Write each fraction as a decimal.**

**a.** $\frac{3}{5}$ **b.** $\frac{1}{2}$

**Solution** **a.** First write 3 as 3.0, and then divide by 5.

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{3\,0} \\ 0 \end{array}$$

So, $\frac{3}{5} = 0.6$.

**b.** Write 1 as 1.0 and then divide by 2.

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1\,0} \\ 0 \end{array}$$

So, $\frac{1}{2} = 0.5$.

Sometimes it is necessary to add more than one zero at the end of the decimal created from the numerator. One additional zero is added and the division continues each time a division step results in a remainder. This is shown in the following examples.

**Example 3** **Write each fraction as a decimal.**

**a.** $\frac{7}{20}$ **b.** $\frac{3}{25}$

**Solution** **a.** When we try to write $\frac{7}{20}$ as a decimal, with 7 written as 7.0, we get

$$\begin{array}{r} 0.3 \\ 20\overline{)7.0} \\ \underline{6\,0} \\ 1\,0 \end{array}$$

Since the division in this step resulted in a remainder of 10, we add another zero to the dividend, 7.0, and continue dividing.

$$
\begin{array}{r}
0.35 \\
20\overline{)7.00} \\
\underline{6\ 0\phantom{0}} \\
1\ 00 \\
\underline{1\ 00} \\
0
\end{array}
$$

The division has no remainder, so $\frac{7}{20} = 0.35$.

**b.** Begin by rewriting $\frac{3}{25}$ as the division 3.0 ÷ 25.

$$
\begin{array}{r}
0.1 \\
25\overline{)3.0} \\
\underline{2\ 5} \\
5
\end{array}
$$

Since there is a remainder, add another zero and continue dividing.

$$
\begin{array}{r}
0.12 \\
25\overline{)3.00} \\
\underline{2\ 5\phantom{0}} \\
50 \\
\underline{50} \\
0
\end{array}
$$

So, $\frac{3}{25} = 0.12$.

**Note to the Teacher** *At this point, some students may suggest that the two previous examples show a pattern in which for single-digit denominators only one zero is needed for the division, and for double-digit denominators two zeros are needed. While praising them for their awareness, use the following example to show that the pattern was only coincidental.*

**Example 4** **Write $\frac{5}{8}$ as a decimal.**

**Solution** Write $\frac{5}{8}$ as the division 5.0 ÷ 8.

$$
\begin{array}{r}
0.6 \\
8\overline{)5.0} \\
\underline{4\ 8} \\
2
\end{array}
$$

Add another zero to the dividend and continue dividing.

$$\begin{array}{r} 0.62 \\ 8\overline{)5.00} \\ \underline{4\,8}\phantom{0} \\ 20 \\ \underline{16} \\ 4 \end{array}$$

Add another zero and continue dividing.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8}\phantom{00} \\ 20\phantom{0} \\ \underline{16}\phantom{0} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

So, $\frac{5}{8} = 0.625$.

## Terminating and Repeating Decimals

Point out to students that sometimes the division never comes out evenly no matter how many zeros we add.

**Example 5** **Write $\frac{1}{3}$ as a decimal.**

**Solution** Use long division as in Examples 2–4.

$$\begin{array}{r} 0.333\ldots \\ 3\overline{)1.000} \\ \underline{9}\phantom{00} \\ 10\phantom{0} \\ \underline{9}\phantom{0} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

The remainder at each step will always be 1 and the next digit in the quotient will always be 3. So the quotient is 0.333… (the dots mean that the pattern continues indefinitely), which can also be written as $0.\overline{3}$ (the bar over the 3 means that the digit repeats).

**Note to the Teacher** *Have students try additional examples for themselves, such as $\frac{1}{9}$ or $\frac{4}{9}$, which are equivalent to the decimal expressions $0.111\ldots = 0.\overline{1}$ and $0.444\ldots = 0.\overline{4}$, respectively.*

So, in a situation where the division never comes out evenly, the fraction cannot be written as a **terminating decimal** (a decimal that ends). However, in these situations there is always a pattern of digits that eventually repeats. Stress that the pattern may involve more than just a single digit, as occurred in Example 5.

**Example 6** **Use a calculator to write $\frac{1}{11}$ as a decimal.**

**Solution** On a calculator, enter 1 [÷] 11 [=].

The result is the decimal expression 0.09090909…, where the dots indicate the repetition of the pattern. So, $\frac{1}{11} = 0.\overline{09}$.

Expressions such as 0.33333… and 0.090909… are called **repeating** (or *nonterminating*) **decimals** because one or more digits repeat.

Have students use a calculator to explore the pattern of repeating digits for other fractions, such as $\frac{1}{6}$, $\frac{4}{7}$, and $\frac{5}{12}$.