

Measures of Variation

Objective Teach students to find the range of a set of data, including the interquartile range of a set of data.

Note to the Teacher *In Chapter 6, students learned that the mean, median, and mode represent the center of a set of data. You may wish to begin by reviewing these terms. Then point out that these measurements do not fully describe a set of data. For instance, one set of data may be concentrated at or near the mean, while another set of data may be spread out. Yet both sets of data may have the same median or modes. Here is an example of this type of situation.*

Example 1 Two groups of people are standing in line to buy tickets to an amusement park. The first group consists of teenagers whose ages are 16, 16, 14, and 14 years old. The second group is a family whose members are 30, 24, 4, and 2 years old. What is the mean age of each group?

Solution Recall that the mean of a set of data is the sum of the data divided by the number of pieces of data.

$$\text{Mean (first group)} = \frac{16 + 16 + 14 + 14}{4} = \frac{60}{4} \text{ or } 15$$

$$\text{Mean (second group)} = \frac{30 + 24 + 4 + 2}{4} = \frac{60}{4} \text{ or } 15$$

The mean age of each group is the same. However, this does not describe the distribution of the ages in the two groups very well. In the first group, all four of the people are close in age. In the second group, there is a large range of ages.

One measure of variation we use is the **range**. The range describes how a set of data varies.

Definition of Range	The range of a set of numbers is the difference between the greatest and least numbers in the set.
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In the example above, the range of ages in the first group is $16 - 14$ or 2. The range of the ages in the second group is $30 - 2$ or 28.

Note to the Teacher *The data set shown has an even number of members. Notice that both the lower half and upper half do not contain the median. When a data set contains an even number of members, the median is the mean of the two middle numbers, and might not be part of the data set. When a data set has an odd number of members, the median is the number in the middle when the data are arranged in order and is not contained in either the lower half or the upper half.*

Quartiles are used in another measure of variation called the **interquartile range**. The interquartile range is the range of the middle half of a set of numbers. To determine the interquartile range, subtract the lower quartile from the upper quartile. In the case of the quiz scores, the interquartile range is $46 - 32$ or 14.

Since the interquartile range is the range of the middle half of a data set, it ignores very large or small numbers of the set. If a set of data has a small interquartile range, there may be some very large or small numbers in the set, but the data in the middle tend to be concentrated. If a set of data has a large interquartile range, we can conclude that the data in the middle is spread out.

Here's an example that summarizes the measures of variation.

Example 4 Find the range, median, upper and lower quartiles, and the interquartile range for the set of data shown below.

12, 8, 15, 16, 8, 49, 32, 18, 22, 28, 14, 20, 4

Solution First, put the numbers in order from least to greatest.

4, 8, 8, 12, 14, 15, 16, 18, 20, 22, 28, 32, 49

The range is the difference between the least and the greatest number in the set. So, the range is $49 - 4$ or 45.

The median is the number in the middle when the data are arranged in order.

4, 8, 8, 12, 14, 15, 16, 18, 20, 22, 28, 32, 49

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The median is 16.

To find the upper quartile and the lower quartile, divide the data set into its lower and upper halves. Then find the median of the lower and upper halves.

$$\begin{array}{ccccccc} & \textit{lower half} & & & \textit{upper half} & & \\ & \overbrace{4, 8, 8, 12, 14, 15, 16} & & & \overbrace{18, 20, 22, 28, 32, 49} & & \\ & \uparrow & & & \uparrow & & \\ & LQ & & & UQ & & \end{array}$$

The lower quartile is $\frac{8+12}{2}$ or 10 and the upper quartile is

$$\frac{22+28}{2} \text{ or } 25.$$

The interquartile range is $25 - 10$ or 15.

Note to the Teacher *The concepts of upper and lower quartile and interquartile range will probably be new to your students. Therefore, it is beneficial to assign many problems so that they can solidify the concepts of the lesson. Be sure to apply these concepts to real-world situations.*

