

Congruent Triangles

Objective Introduce the concept of congruence, study congruent triangles, and compare the angles and sides of congruent triangles.

Note to the Teacher *In this lesson, students will learn the concept of congruence. Basically, two figures are congruent if they have the same shape and size. This gives the idea of congruence but does not give a careful definition of the term. For example, what precisely does the phrase “the same shape” mean? Also, in what measure (perimeter, area, etc.) do we determine if two figures are the same size?*

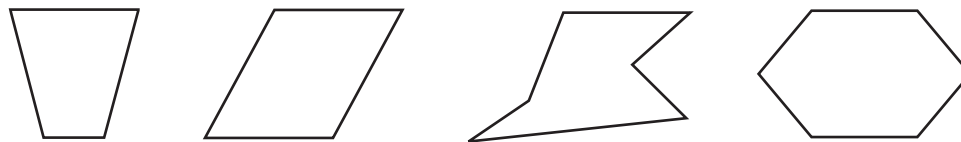
Defining Congruence

Begin with a discussion of how to define the phrase *same size and shape*. Explain to students that polygons are often used when discussing congruence. Then review the definition of polygons.

A **polygon** is a simple closed figure in a plane formed by three or more line segments. The line segments are called sides and their endpoints are called vertices. The sides and vertices have the following properties.

- The sides of a polygon intersect only at the endpoints.
- Each vertex is an endpoint of exactly two sides.

Some examples of polygons are shown.



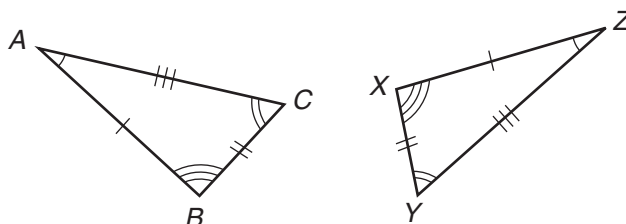
Out of all of the polygons, triangles are probably the most studied. We often study their sides and angles. Two sides of a triangle are **congruent** if their lengths are equal. The expression

$$\overline{AB} \cong \overline{CD}$$

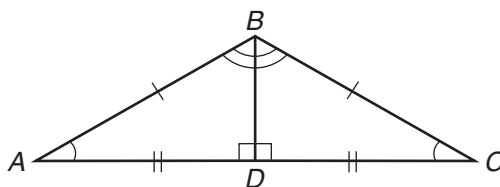
means that segment AB and segment CD are congruent. In the same sense, two angles are congruent if they have equal measures. For triangles and other convex polygons, we can say that two convex polygons are congruent if they have congruent sides and angles.

Congruent Triangles

With regards to the triangles shown, equal angles assure that the polygons have the *same shape*, and the congruent sides assure that the polygons have the *same size*. Since these two criteria are met, the triangles are congruent.



Example 1 The corresponding parts of two congruent triangles are marked in the figure below. Write a congruence statement for the two triangles.



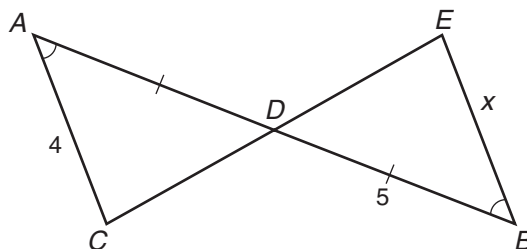
Solution First, list the congruent angles and sides.

$$\begin{array}{ll} \angle A \cong \angle C & \overline{AB} \cong \overline{CB} \\ \angle ABD \cong \angle CBD & \overline{AD} \cong \overline{CD} \\ \angle ADB \cong \angle CDB & \overline{BD} \cong \overline{BD} \end{array}$$

Next, the congruence statement can be written by matching the vertices of the congruent angles.

$$\triangle ADB \cong \triangle CDB$$

Example 2 The triangles in the following figure are congruent. Find the value of x .



Solution In the figure, $\angle ADC$ and $\angle BDE$ are vertical angles. By the Vertical Angle Theorem, we can say $\angle ADC \cong \angle BDE$. It is given that $\angle A \cong \angle B$. So, since the triangles are congruent,

we can conclude that the remaining angles are congruent. That is, $\angle ACD \cong \angle BED$. We know that the sides opposite congruent angles in a congruent triangle are congruent. Since we are given that one side opposite one of the vertical angles at vertex D is equal to 4, we can say that the other side opposite the vertical angle at vertex D is also equal to 4. Therefore, $x = 4$.

Use the following question to help solidify students' understanding of congruence. Ask, "Suppose two triangles are congruent. If one of the triangles has a perimeter of 10 feet, what is the perimeter of the other triangle?" Make sure students understand that if two triangles are congruent, then their corresponding sides are also congruent. The perimeter is the sum of the lengths of the sides. So, two congruent triangles must have equal perimeters. Therefore, the second triangle must have a perimeter of 10 feet.

