Key Concepts

Lesson 23

Squares and Square Roots

Objective Teach students the concept of square root and how to find squares and square roots.

Note to the Teacher *In this lesson, students will work with squares and square roots. In order to find or approximate square roots, students must be familiar with perfect squares. Therefore, it is beneficial to review the squares of the integers through 12.*

Copy the table shown below on the chalkboard. Be sure students recall that the squares of a number and its opposite are equal. That is, $x^2 = (-x)^2$.

x	$-x$	$x^2 = (-x)^2$
0	0	0
1	−1	1
2	−2	4
3	−3	9
4	−4	16
5	−5	25
6	−6	36
7	−7	49
8	−8	64
9	−9	81
10	−10	100
11	−11	121
12	−12	144

Explain to students that finding a square root is an inverse operation to squaring a number. Write the following definition on the board.

Definition of Square Root	The square root of a number is one of its two equal factors. In symbols, if $x^2 = y$, then x is a square root of y.

Explain to students that every positive number has two square roots that are opposites. For example, 2 and −2 are square roots of 4. The number 0 only has one square root, namely 0 itself, because $0^2 = 0$, and $-0 = 0$. Ask, "How many square roots does a negative number have?" **None; the square of any number is positive or 0, and so no number can have its square be a negative number. Therefore a negative number has no square roots.** The symbol $\sqrt{}$, called the **radical sign**, is used to indicate a nonnegative square root. For example, $\sqrt{4} = 2$. In particular, $\sqrt{4}$ is not equal to -2. In order to convey this concept to students, it may be helpful to assign some examples in true and false format as follows.

Exercises

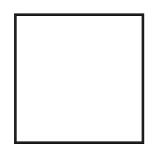
Write *true* or *false*. Explain your reasoning.

1. $\sqrt{25} = 5$ **True; $5^2 = 25$ and 5 is positive.**
2. $-\sqrt{49} = -7$ **True; because $\sqrt{49} = 7$, $7^2 = 49$ and 7 is positive. So, by taking the opposite of both sides, we have $-\sqrt{49} = -7$.**
3. $\sqrt{-49} = -7$ **False; since -49 is negative, it has no square roots. So, $\sqrt{-49}$ is undefined.**

Note to the Teacher *The difference between Exercise 2 and Exercise 3 may be confusing to some students. Explain that we can take the opposite of the square root of a positive number. However, we cannot take the square root of a negative number. The square root of a negative number is undefined.*

The following example shows how square roots are applied in geometry. Do this on the chalkboard.

Example 1 **The area of a square is 144 square inches. What is its perimeter?**



Area = 144 in^2

Solution Let s represent the length of each side. First find the length of each side.

$$
\begin{aligned}
A &= s^2 \\
144 &= s^2 && \textit{Replace A with 144.} \\
\sqrt{144} &= s && \textit{Take the square root of each side.}
\end{aligned}
$$

We need to find the square root of 144. That is, what number s multiplied by itself is 144? Both 12 and -12 when multiplied by themselves are 144. Since length cannot be negative, s must be 12.

Now find the perimeter.

$P = 4 \cdot s$

$P = 4 \cdot 12$ or 48

Therefore, the perimeter of the square is 4(12) or 48 inches.

So far, we have worked with numbers whose square roots are perfect squares. When taking a square root of a number that is not a perfect square, we need to approximate the answer. Consider the following example.

Example 2 **Approximate $\sqrt{125}$.**

Solution Find the perfect squares closest to 125. We know that $121 = 11^2$ and $144 = 12^2$. Therefore, we can conclude that $\sqrt{125}$ is between 11 and 12. Since 125 is closer to 121 than to 144, we can conclude that $\sqrt{125}$ is closer to 11 than to 12. So, we can approximate $\sqrt{125}$ to be about 11.

Note to the Teacher *Have students compare their approximations with the value a calculator gives. If you use a calculator to evaluate $\sqrt{125}$, you get an answer of 11.18033989. So our estimate that $\sqrt{125}$ is about 11 is correct.*