Key Concepts

Lesson 24

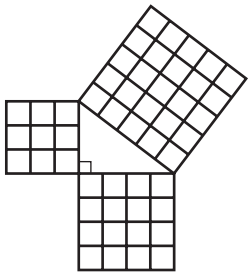
The Pythagorean Theorem

Objective Teach students to use the Pythagorean Theorem to find the length of the side of a right triangle and to solve problems.

Note to the Teacher *The Pythagorean Theorem is probably the most famous theorem in mathematics. Its history dates back to about 2500 years ago to the famous Greek mathematician and philosopher Pythagoras. However, some historians think that the theorem dates back even further as ancient writings have been found that list "Pythagorean triples" or triples of whole numbers that can be the lengths of the sides of a right triangle. You may wish to assign an extra credit project. Have students research the history of the Pythagorean Theorem further and either write a report or give a presentation to the class.*

Modeling the Pythagorean Theorem

It may be helpful to use a model to introduce the Pythagorean Theorem. Begin by drawing the following diagram on the chalkboard or overhead projector.



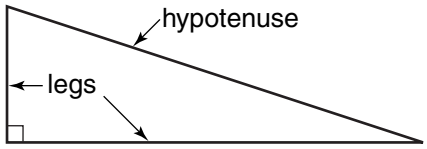
Point out to students that the sides of the right triangle have lengths of 3, 4, and 5 units, and the areas of the squares are 9, 16, and 25 square units, respectively. Explain that these numbers form the basis for the Pythagorean Theorem. The area of the larger square is equal to the total area of the two smaller squares.

$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

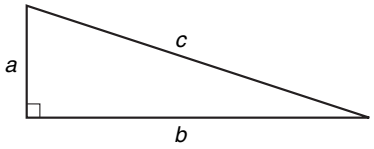
This relationship is true for *any* right triangle.

Before introducing the Pythagorean Theorem in its algebraic form, discuss the following terms.



- The sides of a right triangle that are adjacent to the right angle are called the **legs** of the triangle.
- The side opposite the right angle is called the **hypotenuse**.

Now, state the Pythagorean Theorem.

Pythagorean Theorem	In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. $c^2 = a^2 + b^2$ 

Ask, "How are the model of the Pythagorean Theorem and the expression $c^2 = a^2 + b^2$ related?" **Suppose the lengths of the sides of the right triangle in the model are *a*, *b*, and *c* with *c* being the longest side. Then, the areas of the squares would be a^2, b^2, and c^2, respectively, and c^2 would equal the sum of a^2 and b^2.**

Now is a good time to work through some examples with the class.

Example 1 **The measurements of three sides of a triangle are 5, 6, and 7 units. Determine whether the triangle is a right triangle.**

Solution Use the Pythagorean Theorem to see if the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$c^2 = a^2 + b^2$

$7^2 \stackrel{?}{=} 5^2 + 6^2$ *The hypotenuse is the longest side.*

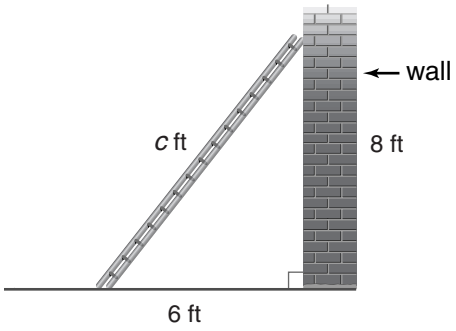
$49 \stackrel{?}{=} 25 + 36$

$49 \neq 61$

Since $7^2 \neq 5^2 + 6^2$, the triangle is not a right triangle.

Example 2 **A ladder is leaning against a wall. Its base is positioned 6 feet from the wall. If the top of the ladder rests against the wall 8 feet from the ground, how long is the ladder? Assume that the wall is at a right angle to the ground.**

Solution First, draw a picture to represent the situation.



Then use the Pythagorean Theorem to solve the problem. The ladder is the hypotenuse of a right triangle whose legs measure 6 feet and 8 feet. Let c represent the length of the ladder.

$c^2 = a^2 + b^2$ *Pythagorean Theorem*

$c^2 = 6^2 + 8^2$ *Replace a with 6 and b with 8.*

$c^2 = 36 + 64$

$c^2 = 100$

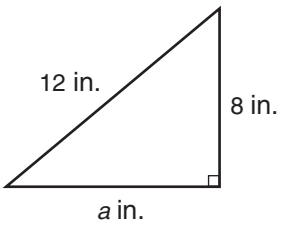
$c = \sqrt{100}$ *Take the square root of each side.*

$c = 10$

So, the length of the ladder is 10 feet.

Sometimes, when working with the Pythagorean Theorem, one has to approximate square roots. Consider the following example.

Example 3 **Find the length of the third side of the right triangle.**



Solution To find the length of the third side, use the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 && \textit{Pythagorean Theorem} \\ 12^2 &= a^2 + 8^2 && \textit{Replace c with 12 and b with 8.} \\ 144 &= a^2 + 64 \\ 144 - 64 &= a^2 && \textit{Subtract 64 from each side.} \\ 80 &= a^2 \\ \sqrt{80} &= a && \textit{Take the square root of each side.} \\ 8.9 &\approx a \end{aligned}$$

The length of the leg is about 8.9 inches.

Note to the Teacher *Give students several problems involving the Pythagorean Theorem as it addresses geometric and algebraic concepts, and practices skills such as taking square roots.*