Key Concepts

Lesson 25

Adding and Subtracting Polynomials

Objective Teach students to add and subtract polynomials.

Note to the Teacher *This lesson is not difficult conceptually, but it is important that students get sufficient practice in addition and subtraction of polynomials.*

Adding Polynomials

Let's use a word problem to introduce addition of polynomials.

Example 1 **Emily and Megan are making square picture frames to sell at an arts and craft show. The cost of each picture frame is determined by its area and perimeter. Megan charges $0.35 for every square inch of area. Emily charges $0.20 for every square inch of area plus an additional $0.25 for every inch of perimeter. Suppose a customer buys two picture frames that are identical in size, one from Megan and one from Emily. What is the total amount spent?**

Solution The length of the side is not given. So, let x represent this value. Now find the cost of the picture frame purchased from Emily. We know that the area of the picture frame is the square of the length of the side. So, its area is given by x^2. Since a square has four sides of equal length, the perimeter is 4 times the length of one side. That is, the perimeter is $4x$. Emily charges $0.20 for every square inch of area plus $0.25 for every inch of perimeter. So, the following expression results.

$$\begin{aligned} 0.2 \times \text{area} + 0.25 \times \text{perimeter} &= 0.2 \times x^2 + 0.25 \times 4x \\ &= 0.2x^2 + x \end{aligned}$$

The cost of the picture frame purchased from Emily is $0.2x^2 + x$ dollars.

Next, find the cost of the picture frame purchased from Megan. She charges $0.35 for every square inch of area. So, the cost is represented by

$$\begin{aligned} 0.35 \times \text{area} &= 0.35 \times x^2 \\ &= 0.35x^2 \end{aligned}$$

The cost of the picture frame purchased from Megan is $0.35x^2$ dollars.

Now, we need to find the total amount spent. Notice that each cost is a polynomial. To find the total cost, we must add the two polynomials together.

$$(0.2x^2 + x) + (0.35x^2) \text{ dollars}$$

Since x^2 occurs twice, we can collect the two expressions into one as follows.

$$\begin{aligned}(0.2x^2 + x) + (0.35x^2) &= (0.2 + 0.35)x^2 + x \\ &= 0.55x^2 + x\end{aligned}$$

So, the total cost of the picture frames is $0.55x^2 + x$ dollars.

When we group together the terms that are similar, we are combining **like terms**. Like terms are expressions that contain the same variables to the same power. Consider the following examples.

The polynomials $x^4 + 4x^3 + 6x^2 + 4x + 1$ and $x^3 + x^2 + 1$ are shown. Notice that there are three pairs of like terms.

$$x^4 + 4x^3 + 6x^2 + 4x + 1 \qquad x^3 + x^2 + 1$$

The polynomials $ab + 2a + 2b$ and $ab - a + 1$ are shown. In this case, there are two pairs of like terms.

$$ab + 2a + 2b \qquad ab - a + 1$$

Whenever we have like terms, we can collect them and add them together to get a single term. For example, in the first diagram above, we can add $4x^3$ and x^3 to get $5x^3$. In the same way, we can add $6x^2$ and x^2 to get $7x^2$, and finally, we can add 1 and 1 to get 2. When we do this, we get the simpler polynomial

$$x^4 + 5x^3 + 7x^2 + 4x + 2.$$

In the second diagram, we can combine the like terms ab and a to get

$$2ab + a + 2b + 1.$$

As you can see, when adding polynomials, it is often necessary to combine like terms. Here's another example.

Example 2 **Find $(x^3 + x + 1) + (3x^3 + x^2 + 2x)$.**

Solution To find the sum, combine the like terms.

$$\begin{aligned}(x^3 + x + 1) + (3x^3 + x^2 + 2x) &= (x^3 + 3x^3) + x^2 + (x + 2x) + 1 \\ &= 4x^3 + x^2 + 3x + 1\end{aligned}$$

So, $(x^3 + x + 1) + (3x^3 + x^2 + 2x) = 4x^3 + x^2 + 3x + 1$.

Key Idea	Sums of polynomials can be simplified by adding like terms. In the same way, differences of polynomials can be simplified by combining like terms.

Example 3 **Find $(x^3 - 3x^2 + 3x - 1) - (x^2 + 2x + 1)$.**

Solution First rewrite the expression as an addition expression.

$$(x^3 - 3x^2 + 3x - 1) + (-x^2 - 2x - 1)$$

Then group all pairs of like terms and simplify.

$$\begin{aligned}&(x^3 - 3x^2 + 3x - 1) + (-x^2 - 2x - 1) \\ &\quad = x^3 + (-3x^2 - x^2) + (3x - 2x) + (-1 - 1) \\ &\quad = x^3 + (-4x^2) + (x) + (-2) \\ &\quad = x^3 - 4x^2 + x - 2\end{aligned}$$

So, $(x^3 - 3x^2 + 3x - 1) - (x^2 + 2x + 1) = x^3 - 4x^2 + x - 2$.

There is another way to organize these calculations so that the addition and subtraction of polynomials is similar to the addition and subtraction of whole numbers. Rewrite the polynomials so that they are aligned one over the other with like terms lined up in columns. Then add the coefficients of each term. Consider the following example.

Suppose we want to add the polynomials $4x^3 + 5x^2 + 6x + 7$ and $2x^3 + 3x + 4$. Rewrite the polynomials as if we were adding whole numbers. Notice that we placed a coefficient of 0 for the missing term.

$$\begin{array}{r} 4x^3 + 5x^2 + 6x + 7 \\ \underline{+2x^3 + 0x^2 + 3x + 4} \end{array}$$

Now, add the coefficients of each term.

$$\begin{array}{r} 4x^3 + 5x^2 + 6x + 7 \\ +2x^3 + 0x^2 + 3x + 4 \\ \hline 6x^3 + 5x^2 + 9x + 11 \end{array}$$

The sum of $4x^3 + 5x^2 + 6x + 7$ and $2x^3 + 3x + 4$ is $6x^3 + 5x^2 + 9x + 11$.

Note to the Teacher *Point out that this process works exactly the same way for subtraction, by just subtracting the entries in each column.*