The Family of Parabolas

The general form of a quadratic function is $y = a(x - h)^2 + k$. Changing the values of a, h, and k results in a different parabola in the family of quadratic functions. The parent graph of the family of parabolas is the graph of $y = x^2$.

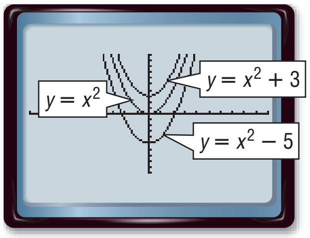
You can use a Casio FX-9750G graphing calculator to analyze the effects that result from changing each of the parameters a, h, and k.

ACTIVITY 1

Graph the set of equations on the same screen in the standard viewing window.

$$y = x^2, y = x^2 + 3, y = x^2 - 5$$

Describe any similarities and differences among the graphs.



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Activity 1 shows how changing the value of k in the equation $y = a(x - h)^2 + k$ *translates* the parabola along the y-axis. If $k > 0$, the parabola is translated k units up, and if $k < 0$, it is translated k units down.

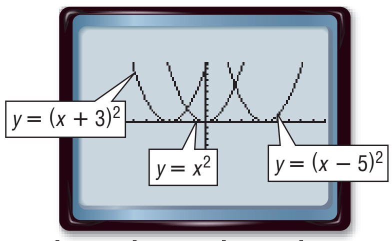
How do you think changing the value of h will affect the graph of $y = (x - h)^2$ as compared to the graph of $y = x^2$?

ACTIVITY 2

Graph the set of equations on the same screen in the standard viewing window.

$$y = x^2, y = (x + 3)^2, \ y = (x - 5)^2$$

Describe any similarities and differences among the graphs.



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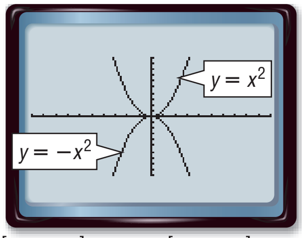
Activity 2 shows how changing the value of h in the equation $y = a(x - h)^2 + k$ *translates* the graph horizontally. If $h > 0$, the graph translates to the right h units. If $h < 0$, the graph translates to the left $|h|$ units.

ACTIVITY 3

Graph each set of equations on the same screen. Describe any similarities and differences among the graphs.

a. $y = x^2, y = -x^2$

The graphs have the same vertex and the same shape. However, the graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down.



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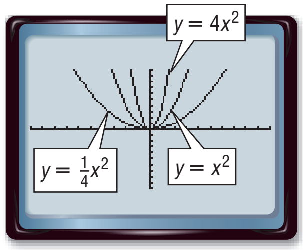
b. $y = x^2, y = 4x^2, y = \frac{1}{4}x^2$

The graphs have the same vertex, (0, 0), but each has a different shape. The graph of $y = 4x^2$ is narrower than the graph of $y = x^2$. The graph of $y = \frac{1}{4}x^2$ is wider than the graph of $y = x^2$.



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Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph. If $a > 0$, the graph opens up, and if $a < 0$, the graph opens down or is *reflected* over the x-axis. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$. Thus, a change in the absolute value of a results in a *dilation* of the graph of $y = x^2$.

ANALYZE THE RESULTS

Consider $y = a(x - h)^2 + k$ where $a \neq 0$.

1. How does changing the value of h affect the graph? Give an example.
2. How does changing the value of k affect the graph? Give an example.
3. How does using $-a$ instead of a affect the graph? Give an example.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

4. $y = x^2, y = x^2 + 2.5$
5. $y = -x^2, y = x^2 - 9$
6. $y = x^2, y = 3x^2$
7. $y = x^2, y = -6x^2$
8. $y = x^2, y = (x + 3)^2$
9. $y = -\frac{1}{3}x^2, y = -\frac{1}{3}x^2 + 2$
10. $y = x^2, y = (x - 7)^2$
11. $y = x^2, y = 3(x + 4)^2 - 7$
12. $y = x^2, y = -\frac{1}{4}x^2 + 1$
13. $y = (x + 3)^2 - 2, y = (x + 3)^2 + 5$
14. $y = 3(x + 2)^2 - 1,$ $y = 6(x + 2)^2 - 1$
15. $y = 4(x - 2)^2 - 3,$ $y = \frac{1}{4}(x - 2)^2 - 1$