

Solving Radical Equations and Inequalities

You can use a Sharp EL-9900C graphing calculator to solve radical equations and inequalities. One way to do this is by rewriting the equation or inequality so that one side is 0 and then using the zero feature on the calculator.

ACTIVITY 1 Solve $\sqrt{x} + \sqrt{x+2} = 3$.

Step 1 Rewrite the equation.

- Subtract 3 from each side of the equation to obtain $\sqrt{x} + \sqrt{x+2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x+2} - 3$ in the Y= list.

KEYSTROKES: Review entering a function in your textbook.

Step 2 Use a table.

- You can use the TABLE function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

KEYSTROKES: **2ndF** **TBLSET**

X	Y1
0	-1.5858
1	-0.2679
2	0.41421
3	0.96812
4	1.44949
5	1.88182

Step 3 Estimate the solution.

- Complete the table and estimate the solution(s).

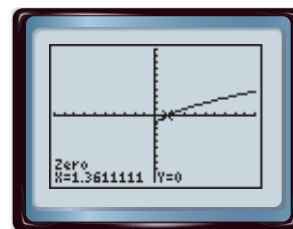
KEYSTROKES: **TABLE**

Since the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

Step 4 Use the zero feature.

- Graph, then select zero from the CALC menu.

KEYSTROKES: **GRAPH** **2nd** **[CALC]** **2**



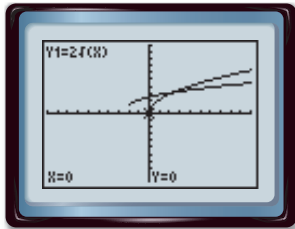
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Place the cursor to the left of the zero and press **ENTER** for the Left Bound. Then place the cursor to the right of the zero and press **ENTER** for the Right Bound. Press **ENTER** to solve. The solution is about 1.36. This agrees with the estimate made by using the TABLE.

ACTIVITY 2 Solve $2\sqrt{x} > \sqrt{x+2} + 1$.

Step 1 Graph each side of the inequality and use the trace feature.

- In the $Y=$ list, enter $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$. Then press **GRAPH**.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

- Press **TRACE**. You can use **▲** or **▼** to switch the cursor between the two curves.

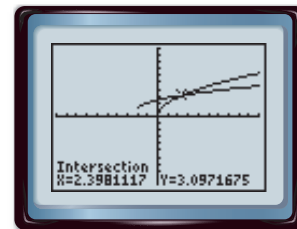
The calculator screen above shows that, for points to the left of where the curves cross, $Y_1 < Y_2$ or $2\sqrt{x} < \sqrt{x+2} + 1$. To solve the original inequality, you must find points for which $Y_1 > Y_2$. These are the points to the right of where the curves cross.

Step 2 Use the intersect feature.

- You can use the **INTERSECT** feature on the **CALC** menu to approximate the x -coordinate of the point at which the curves cross.

KEYSTROKES: **2nd** [**CALC**] **5**

- Press **ENTER** for each of First curve?, Second curve?, and Guess?.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The calculator screen shows that the x -coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about $x > 2.40$. *Use the symbol $>$ in the solution because the symbol in the original inequality is $>$.*

Step 3 Use the table feature to check your solution.

Start the table at 2 and show x -values in increments of 0.1. Scroll through the table.

KEYSTROKES: **2nd** [**TBLSET**] **2** **ENTER** **.1** **ENTER**
2nd [**TABLE**]

X	Y1	Y2
2	2.8284	2.8284
2.1	2.8983	2.9248
2.2	2.9696	3.0494
2.3	3.0423	3.1736
2.4	3.1167	3.2976
2.5	3.1928	3.4214
2.6	3.2706	3.5448
X=2		

Notice that when x is less than or equal to 2.4, $Y_1 < Y_2$. This verifies the solution $\{x \mid x > 2.40\}$.

EXERCISES

Solve each equation or inequality.

1. $\sqrt{x+4} = 3$

2. $\sqrt{3x-5} = 1$

3. $\sqrt{x+5} = \sqrt{3x+4}$

4. $\sqrt{x+3} + \sqrt{x-2} = 4$

5. $\sqrt{3x-7} = \sqrt{2x-2} - 1$

6. $\sqrt{x+8} - 1 = \sqrt{x+2}$

7. $\sqrt{x-3} \geq 2$

8. $\sqrt{x+3} > 2\sqrt{x}$

9. $\sqrt{x} + \sqrt{x-1} < 4$

10. Explain how you could apply the technique in the first example to solving an inequality.