

# Modeling Data Using Exponential Functions

We are often confronted with data for which we need to find an equation that best fits the information. We can find exponential and logarithmic functions of best fit using a Sharp EL-9900C graphing calculator.

## ACTIVITY


The population per square mile in the United States has changed dramatically over a period of years. The table shows the number of people per square mile for several years.

U.S. Population Density			
Year	People per square mile	Year	People per square mile
1790	4.5	1900	21.5
1800	6.1	1910	26.0
1810	4.3	1920	29.9
1820	5.5	1930	34.7
1830	7.4	1940	37.2
1840	9.8	1950	42.6
1850	7.9	1960	50.6
1860	10.6	1970	57.5
1870	10.9	1980	64.0
1880	14.2	1990	70.3
1890	17.8	2000	80.0

- a. Use a graphing calculator to enter the data and draw a scatter plot that shows how the number of people per square mile is related to the year.

- Step 1** Enter the year into L1 and the people per square mile into L2.

**KEYSTROKES:** See your textbook to review how to enter lists.

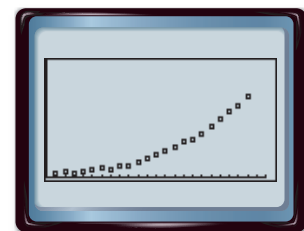
Be sure to clear the Y= list. Use the  key to move the cursor from L1 to L2.

**Source:** Northeast-Midwest Institute

- Step 2** Draw the scatter plot.

**KEYSTROKES:** See your textbook to review how to graph a scatter plot.













Make sure that Plot 1 is on, the scatter plot is chosen, Xlist is L1, and Ylist is L2. Use the viewing window [1780, 2020] with a scale factor of 10 by [0, 115] with a scale factor of 5.



[1780, 2020] scl: 10 by [0, 115] scl: 5

We see from the graph that the equation that best fits the data is a curve. Based on the shape of the curve, try an exponential model.

- Step 3** To determine the exponential equation that best fits the data, use the exponential regression feature of the calculator.

**KEYSTROKES:**   [D] 0 4   L1   
 L2   VARS  [A]   
 [A] 1 

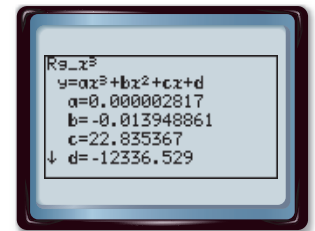
The equation is  $y = 1.835122 \times 10^{-11}(1.014700091)^x$ .

The calculator also reports an  $r$  value of 0.991887235. Recall that this number is a correlation coefficient that indicates how well the equation fits the data. A perfect fit would be  $r = 1$ . Therefore, we can conclude that this equation is a pretty good fit for the data.

To check this equation visually, overlap the graph of the equation with the scatter plot.

**KEYSTROKES:** **GRAPH**

The *residual* is the difference between actual and predicted data. The predicted population per square mile in 2000 using this model was 86.9. (To calculate, press **2nd** **[CALC]** 1 2000 **[ENTER]**.) So, the residual for 2000 was  $80.0 - 86.9$ , or  $-6.9$ .



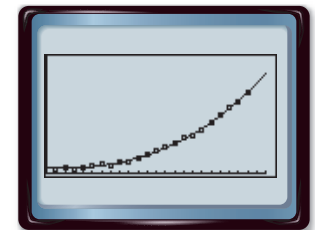
[1780, 2020] scl: 10 by [0, 115] scl: 5

**b. If this trend continues, what will be the population per square mile in 2010?**

To determine the population per square mile in 2010, from the graphics screen, find the value of  $y$  when  $x = 2010$ .

**KEYSTROKES:** **TABLE**

The calculator returns a value of approximately 88.4. If this trend continues, in 2010, there will be approximately 88.4 people per square mile.



[1780, 2020] scl: 10 by [0, 115] scl: 5

## EXERCISES

Jewel received \$30 from her aunt and uncle for her seventh birthday. Her father deposited it into a bank account for her. Both Jewel and her father forgot about the money and made no further deposits or withdrawals. The table shows the account balance for several years.

1. Use a graphing calculator to draw a scatter plot for the data.
2. Calculate and graph the curve of best fit that shows how the elapsed time is related to the balance. Use **ExpReg** for this exercise.
3. Write the equation of best fit.
4. Write a sentence that describes the fit of the graph to the data.
5. Based on the graph, estimate the balance in 41 years. Check this using the **CALC** value.
6. Do you think there are any other types of equations that would be good models for these data? Why or why not?

Elapsed Time (years)	Balance
0	\$30.00
5	\$41.10
10	\$56.31
15	\$77.16
20	\$105.71
25	\$144.83
30	\$198.43