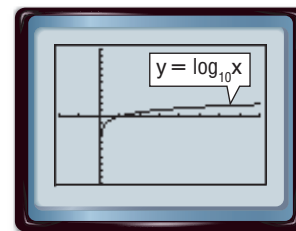


# Solving Logarithmic Equations and Inequalities

You have solved logarithmic equations algebraically. You can also solve logarithmic equations by graphing or by using a table. The Sharp EL-9900C has  $y = \log_{10} x$  as a built-in function. Enter  $\boxed{Y=}$   $\boxed{\text{LOG}}$   $\boxed{X/\theta/T/n}$   $\boxed{\text{GRAPH}}$  to view this graph. To graph logarithmic functions with bases other than 10, you must use the Change of Base Formula,  $\log_a n = \frac{\log_b n}{\log_b a}$ .



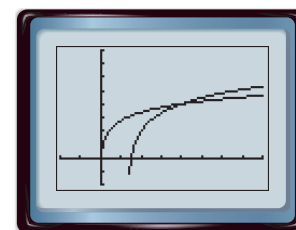
$[-2, 8]$  scl: 1 by  $[-5, 5]$  scl: 1

## ACTIVITY 1 Solve $\log_2(6x - 8) = \log_3(20x + 1)$ .

### Step 1 Graph each side of the equation.

Graph each side of the equation as a separate function. Enter  $\log_2(6x - 8)$  as Y1 and  $\log_3(20x + 1)$  as Y2. Then graph the two equations.

**KEYSTROKES:**  $\boxed{Y=}$   $\boxed{\text{LOG}}$   $\boxed{6}$   $\boxed{X/\theta/T/n}$   $\boxed{-}$   $\boxed{8}$   $\boxed{)}$   $\boxed{\div}$   $\boxed{\text{LOG}}$   $\boxed{2}$   $\boxed{)}$   
 $\boxed{\text{ENTER}}$   $\boxed{\text{LOG}}$   $\boxed{20}$   $\boxed{X/\theta/T/n}$   $\boxed{+}$   $\boxed{1}$   $\boxed{)}$   $\boxed{\div}$   $\boxed{\text{LOG}}$   
 $\boxed{3}$   $\boxed{)}$   $\boxed{\text{GRAPH}}$



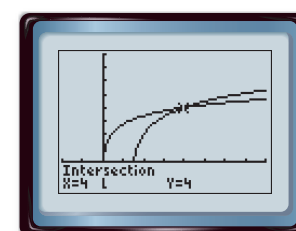
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### Step 2 Use the INTERSECT feature.

Use the intersect feature on the CALC menu to approximate the ordered pair of the point at which the curves cross.

**KEYSTROKES:** See your textbook to review how to use the intersect feature.

The calculator screen shows that the  $x$ -coordinate of the point at which the curves cross is 4. Therefore, the solution of the equation is 4.



$[-2, 8]$  scl: 1 by  $[-2, 8]$  scl: 1

### Step 3 Use the TABLE feature.

**KEYSTROKES:** See your textbook.

Examine the table to find the  $x$ -value for which the  $y$ -values for the graphs are equal. At  $x = 4$ , both functions have a  $y$ -value of 4. Thus, the solution of the equation is 4.

X	Y1	Y2
0	ERR	0
1	ERR	2.7712
2		3.3802
3	3.3219	3.7419
4	4	4
5	4.4594	4.2009
6	4.8074	4.3653

X=0

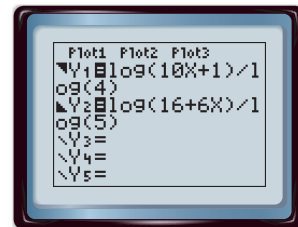
You can use a similar procedure to solve logarithmic inequalities using a graphing calculator.

**ACTIVITY 2**Solve  $\log_4(10x + 1) < \log_5(16 + 6x)$ .**Step 1** Enter the inequalities.

Rewrite the problem as a system of inequalities.

The first inequality is  $\log_4(10x + 1) < y$ , which can be written as  $y > \log_4(10x + 1)$ . Since this inequality includes the *greater than* symbol, shade above the curve.

The second inequality is  $y < \log_5(16 + 6x)$ . Shade below the curve since this inequality contains *less than*.

**Step 2** Graph the system.KEYSTROKES: **GRAPH**

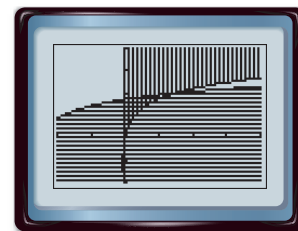
The left boundary of the solution set is where the first inequality is undefined. It is undefined for  $10x + 1 \leq 0$ .

$$10x + 1 \leq 0$$

$$10x \leq -1$$

$$x \leq -\frac{1}{10}$$

Use the calculator's **intersect** feature to find the right boundary. You can conclude that the solution set is  $\{x \mid -0.1 < x < 1.5\}$ .



[-2, 8] scl: 1 by [-2, 8] scl: 1

**Step 3** Use the TABLE feature to check your solution.

Start the table at  $-0.1$  and show  $x$ -values in increments of  $0.1$ . Scroll through the table.

KEYSTROKES: **TABLE**  $-0.1$  **ENTER**.5 **ENTER** **TABLE**

The table confirms the solution of the inequality is  $\{x \mid -0.1 < x < 1.5\}$ .

X	Y1	Y2
ERR:	1.699	
0	1.7227	
.1	.79248	1.7456
.2	1	1.7676
.3	1.161	1.7885
.4	1.2925	1.8085
.5		1.8285

X	Y1	Y2
1.1	1.7297	1.9206
1.2	1.7925	1.9373
1.3	1.8502	1.9536
1.4	1.9027	1.9694
1.5	1.9534	1.9849
1.6	2.0437	2.0147

**EXERCISES**

Solve each equation or inequality. Check your solution.

1.  $\log_2(3x + 2) = \log_3(12x + 3)$

2.  $\log_6(7x + 1) = \log_4(4x - 4)$

3.  $\log_2 3x = \log_3(2x + 2)$

4.  $\log_{10}(1 - x) = \log_5(2x + 5)$

5.  $\log_4(9x + 1) > \log_3(18x - 1)$

6.  $\log_3(3x - 5) \geq \log_3(x + 7)$

7.  $\log_5(2x + 1) < \log_4(3x - 2)$

8.  $\log_2 2x \leq \log_4(x + 3)$