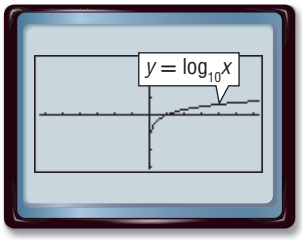
Graphing Technology Lab

Solving Logarithmic Equations and Inequalities

Casio FX-9750G

You have solved logarithmic equations algebraically. You can also solve logarithmic equations by graphing or by using a table. The **Casio FX-9750G** has $y = \log_{10} x$ as a built-in function. Enter MENU 5 LOG X,θ,T EXE F6 to view this graph. To graph logarithmic functions with bases other than 10, you must use the Change of Base Formula, $\log_a n = \frac{\log_b n}{\log_b a}$.



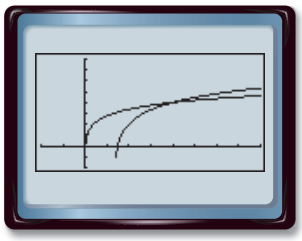
ACTIVITY 1

Solve $\log_2 (6x - 8) = \log_3 (20x + 1)$.

Step 1 Graph each side of the equation.

Graph each side of the equation as a separate function. Enter $\log_2 (6x - 8)$ as **Y1** and $\log_3 (20x + 1)$ as **Y2**. Then graph the two equations.

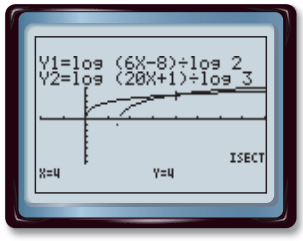
KEYSTROKES: MENU 5 LOG (6 X,θ,T − 8) ÷ LOG (2) EXE LOG (20 X,θ,T + 1) ÷ LOG (3) EXE F6



Step 2 Use the **intersect** feature.

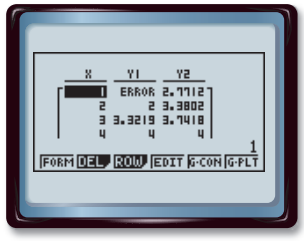
Use the **intersect** feature on the **G-Solv** menu to approximate the ordered pair of the point at which the curves intersect.

The calculator screen shows that the x-coordinate of the point at which the curves intersect is 4. Therefore, the solution of the equation is 4.



Step 3 Use the **TABLE** feature.

Examine the table to find the x-value for which the y-values for the graphs are equal. At $x = 4$, both functions have a y-value of 4. Thus, the solution of the equation is 4.

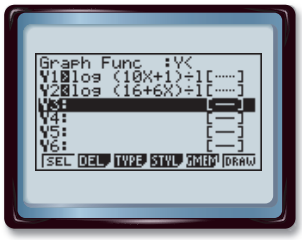


You can use a similar procedure to solve logarithmic inequalities using a graphing calculator.

ACTIVITY 2

Solve $\log_4 (10x + 1) < \log_5 (16 + 6x)$.

Step 1 Enter the inequalities.



Rewrite the problem as a system of inequalities.

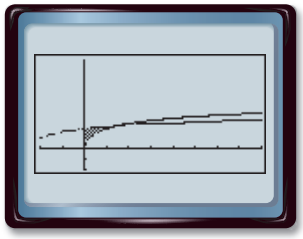
The first inequality is $\log_4 (10x + 1) < y$ or $y > \log_4 (10x + 1)$. Since this inequality includes the *greater than* symbol, shade above the curve.

To show that an equation is greater than or equal to, use F3 F6 F1.
To show that an equation is less than or equal to, use F3 F6 F2.

The second inequality is $y < \log_5 (16 + 6x)$. Shade below the curve since this inequality contains *less than*.

KEYSTROKES: MENU 5 LOG (10 X,θ,T + 1) ÷ LOG (4) EXE
LOG (16 + 6 X,θ,T) ÷ LOG (5) EXE F6

Step 2 Graph the system.



The left boundary of the solution set is where the first inequality is undefined. It is undefined for $10x + 1 \leq 0$.

$$10x + 1 \leq 0$$
$$10x \leq -1$$
$$x \leq -\frac{1}{10}$$

Use the calculator's **intersect** feature to find the right boundary. You can conclude that the solution set is $\{x \mid -0.1 < x < 1.5\}$.

Step 3 Use the **TABLE** feature to check your solution.

Start the table at -0.1 and show x-values in increments of 0.1. Scroll through the table.

The table confirms the solution of the inequality is $\{x \mid -0.1 < x < 1.5\}$.

Exercises

Solve each equation or inequality. Check your solution.

1. $\log_2 (3x + 2) = \log_3 (12x + 3)$

2. $\log_6 (7x + 1) = \log_4 (4x - 4)$

3. $\log_2 3x = \log_3 (2x + 2)$

4. $\log_{10} (1 - x) = \log_5 (2x + 5)$

5. $\log_4 (9x + 1) > \log_3 (18x - 1)$

6. $\log_3 (3x - 5) \geq \log_3 (x + 7)$

7. $\log_5 (2x + 1) < \log_4 (3x - 2)$

8. $\log_2 2x \leq \log_4 (x + 3)$