

# Simplifying Rational Expressions (Pages 638–643)



A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. Any values of the variable that result in a denominator of zero must be excluded from the domain of the variable. These are called **excluded values** of the rational expression, and can be investigated on the graph of the related **rational function**. To simplify a rational expression, factor the numerator and denominator, and then divide by the GCF.

## EXAMPLES

**A** Find the excluded value(s) for  $\frac{2x}{x(x+5)}$ .

Exclude the values for which  $x(x+5) = 0$ .

$$x(x+5) = 0$$

$$x = 0 \text{ or } x + 5 = 0$$

$$x = -5$$

So,  $x$  cannot equal 0 or  $-5$

**B** Simplify  $\frac{b-3}{b^2-2b-3}$ .

$$\frac{b-3}{b^2-2b-3} = \frac{b-3}{(b-3)(b+1)}$$

Factor the denominator.

$$= \frac{\overset{1}{\cancel{b-3}}}{\underset{1}{(b-3)}(b+1)}$$

Simplify the fraction by dividing by the GCF,  $b-3$ .

$$= \frac{1}{b+1}$$

Note that  $-1$  and  $3$  are excluded values for  $b$ .

### Try These Together

**1.** Find the excluded value(s) for  $\frac{7a^3}{14a}$ .

HINT: Exclude values for which  $14a = 0$ .

**2.** Simplify  $\frac{x^2+3x+2}{x^2-4x-5}$ .

HINT: Factor both the numerator and the denominator.

## PRACTICE

Find the excluded value(s) for each rational expression.

**3.**  $\frac{8a}{a(a+3)}$

**4.**  $\frac{x^2-25}{x-5}$

**5.**  $\frac{3x+3}{x^2-1}$

**6.**  $\frac{a+7}{a^2+9a+14}$

Simplify each rational expression.

**7.**  $\frac{6x^2y}{30x}$

**8.**  $\frac{9x^4y^2z}{x^6y}$

**9.**  $\frac{20xyz^3}{60x^2yz^3}$

**10.**  $\frac{10x-5x^2}{2x^2}$

**11.**  $\frac{12x}{3x+6}$

**12.**  $\frac{b^2-4}{4b-8}$

**13.**  $\frac{x^2+6x+8}{6x+24}$

**14.**  $\frac{y^2+7y+6}{y^2+5y-6}$



**15. Standardized Test Practice** Simplify the rational expression  $\frac{2x^2-98}{8x-56}$ .

**A**  $4(x+7)$

**B**  $4(x-7)$

**C**  $\frac{x^2-49}{x-7}$

**D**  $\frac{x+7}{4}$

Answers: 1. 0,  $\frac{x-5}{x+2}$  2.  $\frac{x-5}{x+2}$  3. 0, -3 4. 5, -1, 1 5. -1, 1 6. -7, -2 7.  $\frac{5}{xy}$  8.  $\frac{x^2}{9yz}$  9.  $\frac{3x}{1}$  10.  $\frac{2x}{10-5x}$  11.  $\frac{x+2}{4x}$  12.  $\frac{b+2}{4}$  13.  $x+2$  14.  $\frac{y-1}{y+1}$  15. D

# Multiplying and Dividing Rational Expressions

(Pages 644–649)



To multiply rational expressions, you can divide by the common factors either *before* or *after* you multiply the expressions. To divide algebraic rational expressions, multiply by the reciprocal of the divisor (the second fraction).

## EXAMPLES

**A** Find  $\frac{2x^2(3x-2)}{3x^2+x-2} \cdot \frac{1}{4x}$ .

$$\begin{aligned} \frac{2x^2(3x-2)}{3x^2+x-2} \cdot \frac{1}{4x} &= \frac{2x^2(3x-2)}{(3x-2)(x+1)} \cdot \frac{1}{4x} && \text{Factor.} \\ &= \frac{\overset{1}{x} \cancel{2x^2(3x-2)}}{\underset{1}{(3x-2)}(x+1)} \cdot \frac{1}{\underset{1}{2} \cancel{4x}} && \text{Divide by the GCF,} \\ &= \frac{x}{2(x+1)} \text{ or } \frac{x}{2x+2} && \text{Multiply. Then, simplify.} \end{aligned}$$

**B** Find  $\frac{x^2-4}{5x} \div \frac{x+2}{x-2}$ .

$$\begin{aligned} \text{The reciprocal of } \frac{x+2}{x-2} \text{ is } \frac{x-2}{x+2}. \\ \frac{x^2-4}{5x} \div \frac{x+2}{x-2} &= \frac{x^2-4}{5x} \cdot \frac{x-2}{x+2} \\ &= \frac{\overset{1}{\cancel{(x+2)}}(x-2)}{5x} \cdot \frac{x-2}{\cancel{x+2}} && \text{Factor. Then divide by } x+2. \\ &= \frac{(x-2)(x-2)}{5x} \text{ or } \frac{x^2-4x+4}{5x} && \text{Multiply.} \end{aligned}$$

## Try These Together

**1.** Find  $\frac{ab^2}{12} \cdot \frac{6}{b}$ .

*HINT: Divide both numerator and denominator by the same quantity—their greatest common factor.*

**2.** Find  $\frac{3a-15}{a+4} \div (a-5)$ .

*HINT: The reciprocal of  $a-5$  is  $\frac{1}{a-5}$ .*

## PRACTICE

Find each product.

**3.**  $\frac{15a}{b^3} \cdot \frac{2b^4}{3}$

**4.**  $\frac{25mr^2}{4n} \cdot \frac{10r^3}{5m}$

**5.**  $\frac{x^2+2x-15}{x^2+4x} \cdot \frac{x^2}{x+5}$

**6.**  $\frac{y^2-36}{y+3} \cdot \frac{y-4}{y^2+2y-24}$

**7.**  $\frac{3x+12}{x^2-x-2} \cdot \frac{2x-2}{6x+24}$

**8.**  $\frac{12(a-1)}{3a} \cdot \frac{a^2}{a-1}$

Find each quotient.

**9.**  $\frac{8x}{3yz^2} \div \frac{4xy}{3yz}$

**10.**  $10bc^2 \div \frac{2abc}{8b}$

**11.**  $\frac{x-8}{x+3} \div \frac{x+2}{x+2}$

**12.**  $\frac{b^2-25}{4} \div (b+5)$

**13.**  $\frac{2k+10}{k-3} \div \frac{2}{k-3}$

**14.**  $\frac{x+1}{x^2+2x+1} \div \frac{x-3}{x+1}$



**15. Standardized Test Practice** Find the quotient  $\frac{x+1}{2} \div \frac{x^2+6x+5}{4}$ .

**A**  $\frac{2}{x+5}$

**B**  $2(x+5)$

**C**  $\frac{1}{2}(x+5)$

**D**  $\frac{x+5}{2}$

Answers: 1.  $\frac{ab}{3}$  2.  $\frac{a+4}{3}$  3.  $10ab$  4.  $\frac{2}{25n^4}$  5.  $\frac{x(x+4)}{(x-3)}$  6.  $\frac{y+3}{y-6}$  7.  $\frac{(x+1)(x-2)}{x-1}$  8.  $4a$  9.  $\frac{yz}{2}$  10.  $\frac{a}{40bc}$  11.  $\frac{x+3}{x-8}$  12.  $\frac{b-5}{4}$  13.  $k+5$  14.  $\frac{x-3}{1}$  15. A

# Dividing Polynomials (Pages 650–655)

In the previous lesson, you learned that some divisions can be performed using factoring. You can also divide polynomials using long division.

## EXAMPLES

**A** Find  $(3x^2 - 7x - 6) \div (3x + 2)$ .

*Step 1: Divide the first term of the dividend,  $3x^2$ , by the first term of the divisor,  $3x$ .*

$$\begin{array}{r} x \\ 3x + 2 \overline{) 3x^2 - 7x - 6} \\ \underline{(-) 3x^2 + 2x} \phantom{- 6} \\ -9x - 6 \phantom{- 6} \\ \phantom{- 9x} \underline{- 6} \\ 0 \end{array} \quad \begin{array}{l} 3x^2 \div 3x = x \\ \text{Multiply } x \text{ and } 3x + 2. \\ \text{Subtract.} \end{array}$$

*Step 2: Divide the first term of the partial dividend,  $-9x$ , by the first term of the divisor,  $3x$ .*

$$\begin{array}{r} x - 3 \\ 3x + 2 \overline{) 3x^2 - 7x - 6} \\ \underline{(-) 3x^2 + 2x} \phantom{- 6} \\ -9x - 6 \\ \underline{(-) -9x - 6} \\ 0 \end{array} \quad \begin{array}{l} \text{Bring down } -6; \\ -9x \div 3x = -3 \\ \text{Multiply } -3 \text{ and } 3x + 2. \\ \text{Subtract.} \end{array}$$

So,  $(3x^2 - 7x - 6) \div (3x + 2) = x - 3$ .

**B** Find  $(t^2 - 5t + 10) \div (t + 3)$ .

$$\begin{array}{r} t \\ t + 3 \overline{) t^2 - 5t + 10} \\ \underline{(-) t^2 + 3t} \phantom{+ 10} \\ -8t \phantom{+ 10} \\ \phantom{- 8t} \underline{+ 10} \\ 34 \end{array} \quad \begin{array}{l} t^2 \div t = t \\ \text{Multiply } t \text{ and } t + 3. \\ \text{Subtract.} \\ t - 8 \\ \text{Bring down } 10; \\ -8t \div t = -8 \\ \text{Multiply } -8 \text{ and } t + 3. \\ \text{Subtract.} \end{array}$$

The quotient is  $t - 8$  with remainder 34 or  $t - 8 + \frac{34}{t + 3}$ .

## Try These Together

1. Find  $(x^2 + 4x - 8) \div (x + 2)$ .

2. Find  $(y^2 + 7y + 10) \div (y + 2)$ .

*HINT: Begin by dividing the first term of the dividend by the first term of the divisor.*

## PRACTICE

Find each quotient.

- $(k^2 - 12k + 27) \div (k - 3)$
- $(x^2 - 5x + 6) \div (x - 3)$
- $(2y^2 + 10y + 8) \div (y + 4)$
- $(2b^2 - 5b + 8) \div (b - 2)$
- $(t^2 - 10t + 16) \div (t - 8)$
- $(2x^2 + 13x + 6) \div (2x + 1)$
- $\frac{y^3 - 4y^2 + 2y + 8}{y + 1}$
- $(x^2 + 7x + 10) \div (x + 2)$
- $(a^2 - 3a - 4) \div (a + 1)$
- $(x^2 + 8x + 14) \div (x + 1)$
- $(2x^2 + 9x + 3) \div (x + 3)$
- $(2n^2 + 7n + 3) \div (n + 3)$
- $(6x^2 + x - 15) \div (2x - 3)$
- $\frac{x^3 + x - 2}{x - 1}$



17. **Standardized Test Practice** Find  $(3x^2 + 6x + 9) \div (x + 3)$ .

A  $3x - 3$

B  $3x + 2$

C  $3x + 2 + \frac{9}{x + 3}$

D  $3x - 3 + \frac{18}{x + 3}$

Answers: 1.  $x + 2 - \frac{x + 3}{6}$  2.  $y + 5$  3.  $k - 9$  4.  $x + 5$  5.  $x - 2$  6.  $a - 4$  7.  $2y + 2$  8.  $x + 7 + \frac{x + 1}{7}$  9.  $2b - 1 + \frac{b - 2}{6}$  10.  $2x + 3 - \frac{x + 3}{6}$  11.  $t - 2$  12.  $2n + 1$  13.  $x + 6$  14.  $3x + 5$  15.  $y^2 - 5y + 7 + \frac{y + 1}{1}$  16.  $x^2 + x + 2$  17. D

# Combining Rational Expressions with Like Denominators

(Pages 656–661)



To add or subtract rational expressions with like denominators, add or subtract the numerators and then write the sum or difference over the common denominator. Remember to simplify your answer, if necessary, by dividing by the GCF.

## EXAMPLES

**A** Find  $\frac{7t}{9} - \frac{2t-1}{9}$ .

$$\begin{aligned}\frac{7t}{9} - \frac{2t-1}{9} &= \frac{7t - (2t-1)}{9} \\ &= \frac{7t - 2t + 1}{9} \\ &= \frac{5t + 1}{9}\end{aligned}$$

**B** Find  $\frac{2a}{2a-1} + \frac{2a-2}{2a-1}$ .

Sometimes you must factor to simplify the sum or difference.

$$\begin{aligned}\frac{2a}{2a-1} + \frac{2a-2}{2a-1} &= \frac{2a + (2a-2)}{2a-1} \\ &= \frac{4a-2}{2a-1} \\ &= \frac{2(2a-1)}{2a-1} && \text{Factor the numerator.} \\ &= \frac{2\cancel{(2a-1)}}{\cancel{2a-1}} \\ &= 2\end{aligned}$$

## PRACTICE

Find each sum or difference. Write in simplest form.

1.  $\frac{9}{3m} + \frac{-12}{3m}$

2.  $\frac{12x}{21} - \frac{5x}{21}$

3.  $\frac{3}{x} - \frac{9}{x}$

4.  $\frac{t+2}{4} - \frac{t}{4}$

5.  $\frac{y+3}{2} + \frac{4y-6}{2}$

6.  $\frac{2x}{8} - \frac{-14x}{8}$

7.  $\frac{3c}{4c+1} + \frac{c+1}{4c+1}$

8.  $\frac{7k}{k+2} - \frac{6k}{k+2}$

9.  $\frac{x-3}{x-5} - \frac{2}{x-5}$

10.  $\frac{3n}{2n-3} + \frac{n-6}{2n-3}$

11.  $\frac{3d-2}{2} + \frac{d+4}{2}$

12.  $\frac{a}{a+4} - \frac{8+a}{a+4}$

13.  $\frac{2n}{5n+5} - \frac{n-1}{5n+5}$

14.  $\frac{x+2}{x-1} + \frac{2x-5}{x-1}$

15.  $\frac{x-9}{x+2} - \frac{2x-12}{x+2}$



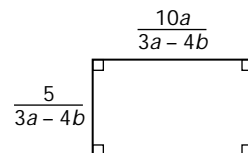
- 16. Standardized Test Practice** Which of the following is an expression for the perimeter of the rectangle?

A  $\frac{15a}{3a-4b}$

B  $\frac{20a+10}{3a-4b}$

C  $\frac{15a}{9a-8b}$

D  $\frac{10a+5}{3a-4b}$



Answers: 1.  $-\frac{m}{1}$  2.  $\frac{3}{x}$  3.  $-\frac{6}{x}$  4.  $\frac{1}{2}$  5.  $\frac{5y-3}{2}$  6.  $2x$  7. 1 8.  $\frac{k+2}{k}$  9. 1 10. 2 11.  $2d+1$  12.  $-\frac{a+4}{8}$  13.  $\frac{5}{1}$  14. 3 15.  $-\frac{x+3}{2}$  16. B



## Solving Rational Equations

(Pages 668–673)



A **rational equation** is an equation that contains at least one rational expression. There are three steps in solving rational equations.

**Step 1** Find the LCD of each term.

**Step 2** Multiply each side of the equation by the LCD.

**Step 3** Use the Distributive Property to simplify.

## EXAMPLE

Solve  $\frac{a}{a+1} + \frac{3a}{a+1} = 3$ .

$$\frac{a}{a+1} + \frac{3a}{a+1} = 3 \quad \text{The LCD is } a + 1.$$

$$(a + 1)\left(\frac{a}{a+1} + \frac{3a}{a+1}\right) = (a + 1)3 \quad \text{Multiply each side by the LCD.}$$

$$(a + 1)\frac{a}{a+1} + (a + 1)\frac{3a}{a+1} = (a + 1)3 \quad \text{Use the Distributive Property.}$$

$$a + 3a = 3a + 3$$

$$4a = 3a + 3$$

$$a = 3 \quad \text{Subtract } 3a \text{ from each side}$$

Check:  $\frac{a}{a+1} + \frac{3a}{a+1} = 3$

$$\frac{3}{4} + \frac{9}{4} \stackrel{?}{=} 3 \quad \text{Replace } a \text{ with } 3.$$

$$\frac{12}{4} \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

## PRACTICE

Solve each equation. Check your solution.

1.  $\frac{2}{3y} + \frac{4}{y} = \frac{1}{3}$

2.  $\frac{n-4}{n} = \frac{5}{n}$

3.  $\frac{14}{x} - 4 = \frac{6}{x}$

4.  $\frac{1}{t} = \frac{3}{t-6}$

5.  $\frac{y-2}{3} - 5 = -\frac{4y}{5}$

6.  $\frac{h+2}{h} - \frac{6-h}{h} = 3$

7.  $\frac{k+8}{k} - \frac{k-4}{k} = 3$

8.  $\frac{a+1}{a} = \frac{a+1}{a-4}$

9.  $\frac{n}{n-1} + \frac{2n}{n-1} = 2$

10.  $\frac{6}{b-7} + \frac{7}{b} = \frac{6}{b}$

11.  $\frac{5}{c+4} - \frac{2}{c} = \frac{1}{c}$

12.  $\frac{4}{s-1} + \frac{1}{s+1} = \frac{1}{s^2-1}$

13.  $\frac{1}{3p} - \frac{2}{p-2} = -\frac{3}{p}$

14.  $\frac{5}{y+5} + \frac{2}{y} = \frac{2}{y+5}$

15.  $\frac{n-1}{n} = \frac{n+1}{n+3}$



16. **Standardized Test Practice** Solve  $\frac{3}{2j} - \frac{4}{3j} = \frac{1}{3}$ .

A -1

B  $-\frac{1}{2}$

C  $\frac{1}{2}$

D  $\frac{1}{6}$

Answers: 1. 14 2. 9 3. 2 4. -3 5. 5 6. -4 7. 4 8. -1 9. -2 10. 1 11. 6 12.  $-\frac{5}{2}$  13. 5 14. -2 15. 3 16. C

## Chapter 15 Review



## Connect the Dots

Imagine that you have just won the vacation of a lifetime in a raffle. Complete this puzzle to find out how you will be traveling to your destination. First simplify each expression completely. Then connect the dots following the instructions in the box at the right.

1.  $\frac{9x^2}{3xy}$

2.  $\frac{x^2 + 5x}{3x + 15}$

3.  $\frac{x-2}{2} \cdot \frac{x+4}{x^2-4}$

4.  $\frac{x^2-x}{x^2-1} \div \frac{x}{x+1}$

5.  $(x^3 + 5x^2 + 5x - 3) \div (x + 3)$

6.  $\frac{6x}{x-3} - \frac{18}{x-3}$

7.  $\frac{1}{5x} - \frac{3}{7x}$

8.  $\frac{1}{x+3} + \frac{1}{x-3}$

**Connect the answers to each problem in the following order:**

Connect #1 to #2.

Connect #3 to #4.

Connect #5 to #6.

Connect #2 to #7.

Connect #5 to #3.

Connect #7 to #8.

Connect #4 to #6.

$$\frac{x^2}{(x+1)^2} \quad -\frac{8}{35x} \quad \frac{3}{x}$$

$$\frac{x+5}{3} \quad \frac{x-2}{2} \quad \frac{22}{35x}$$

$$\frac{x^2+3}{(x+3)(x-3)} \quad -\frac{2}{35x}$$

$$\frac{x-2}{2(x-4)} \quad x \quad \frac{x^2+2x-8}{2(x+2)(x-2)}$$

$$\frac{3x}{xy} \quad \frac{3x}{y} \quad \frac{x}{3}$$

$$\frac{x+4}{2(x+2)} \quad \frac{2x}{(x+3)(x-3)} \quad 1$$

$$x^2 + 2x + 1 \quad \frac{x}{3y} \quad x^2 - 2x + 1$$

$$x^2 + 2x - 1 \quad x^2 - 2x - 1 \quad 6$$

Answers are located on page 122.